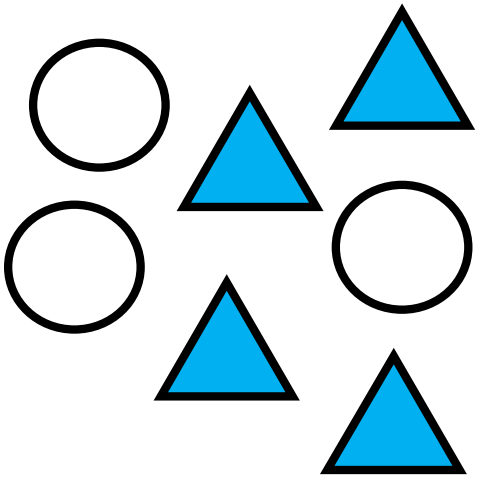
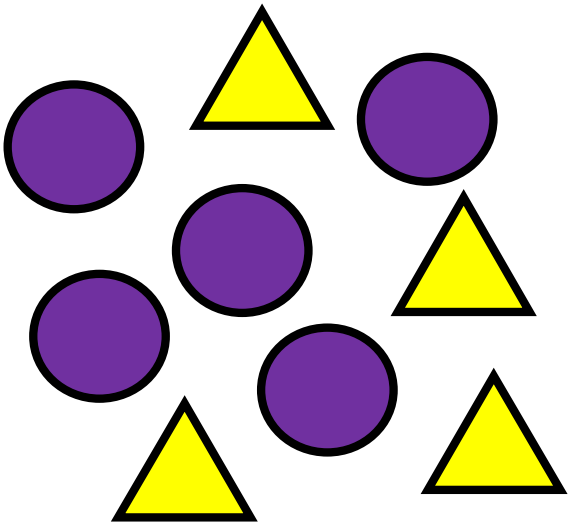
Ratio

a comparison of any two quantities

SET A



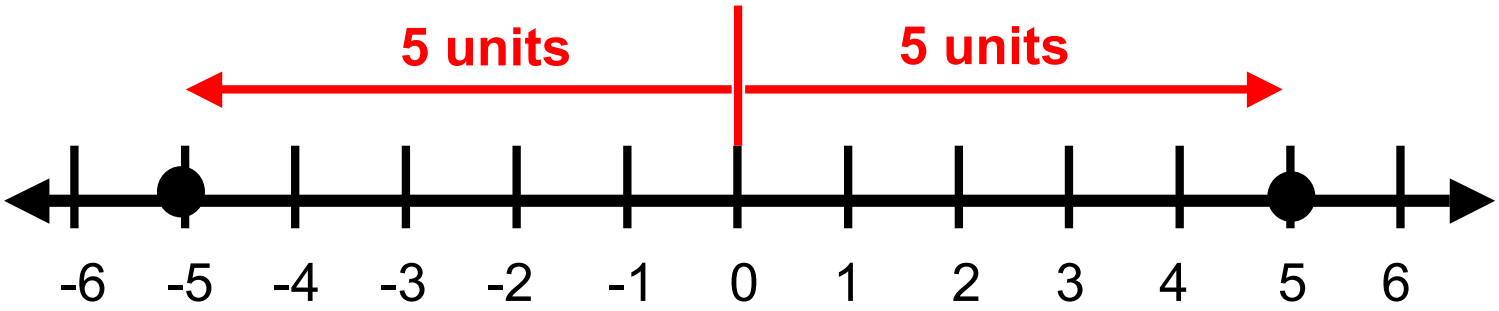
SET B



▲ to ○	**4 to 3**
▲ to all of set A	**4/7**
○ to ●	**3 : 5**
set B to set A	**9 to 7, 9/7, or 9:7**

Absolute Value

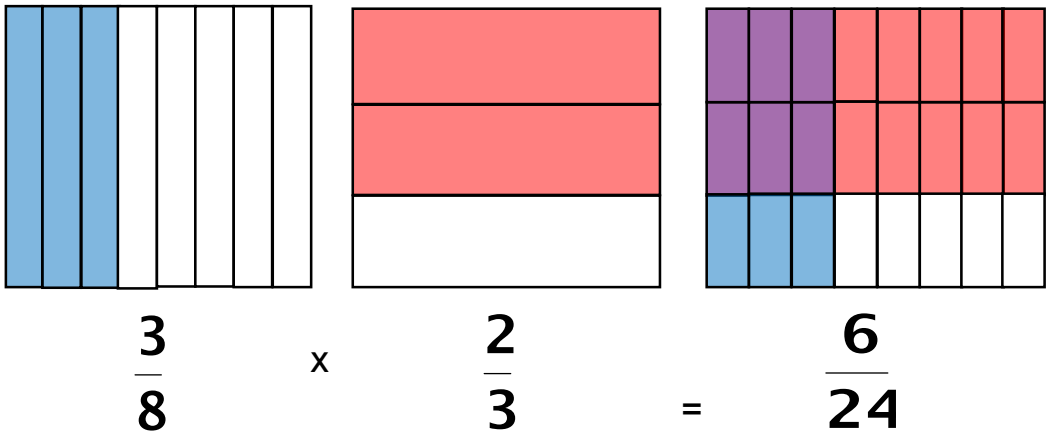
$|5| = 5 \quad |-5| = 5$



distance a number is from zero

Fraction Multiplication

How much is 3/8 of 2/3 ?

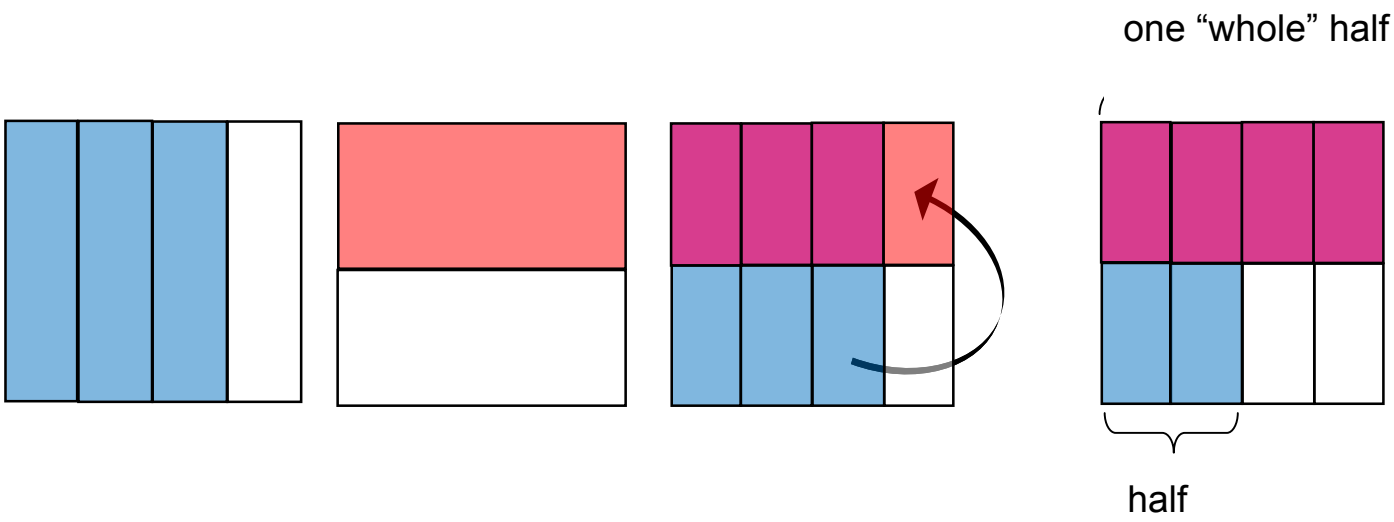


$$3/8 \times 2/3 = 6/24 = 1/4$$

Fraction Division

$3/4 \div 1/2$

How many halves are in three-fourths?



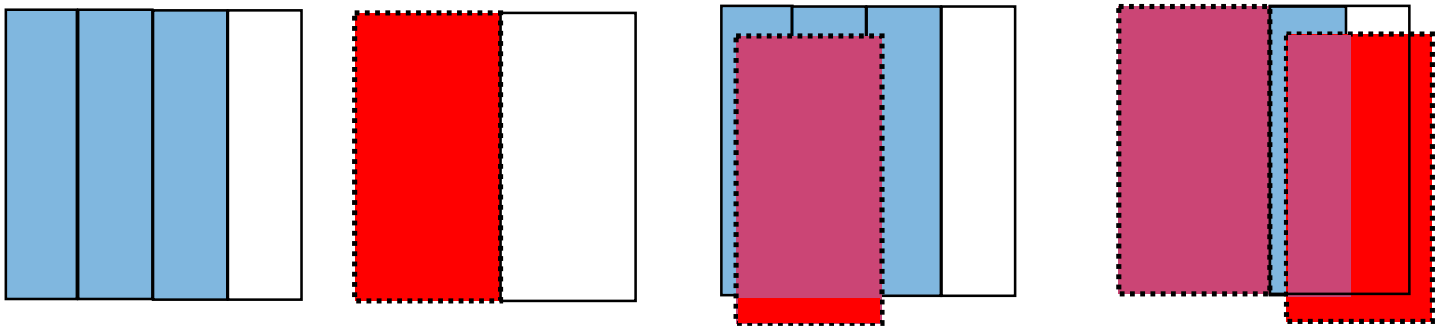
There are $1/12$ halves in three-fourths.

$$3/4 \div 1/2 = 1/12$$

Fraction Division

$3/4 \div 1/2$

How many halves are in three-fourths?

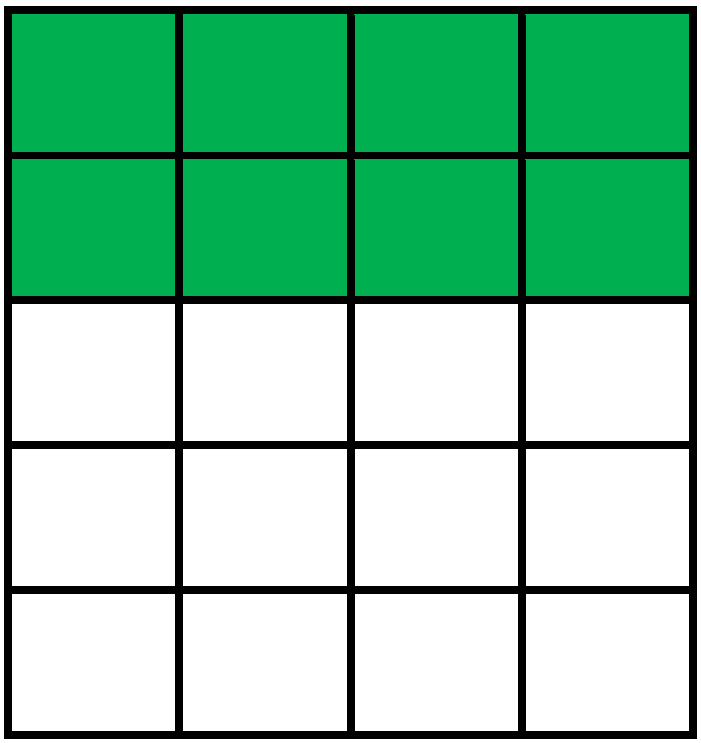


three-fourths one-half 1 “whole” one-half one-half

There are 112 halves in three-fourt

$3/4 \div 1/2 = 1/12$

Equivalent Relationships



Fraction:

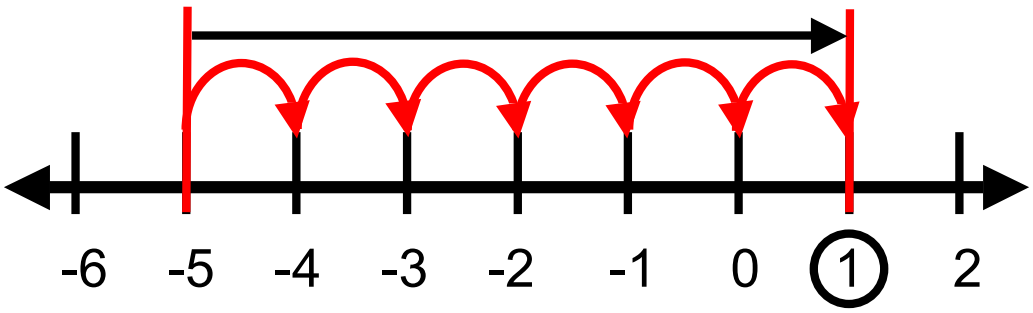
Decimal: 0.4

Percent: 40%

Integer Operations

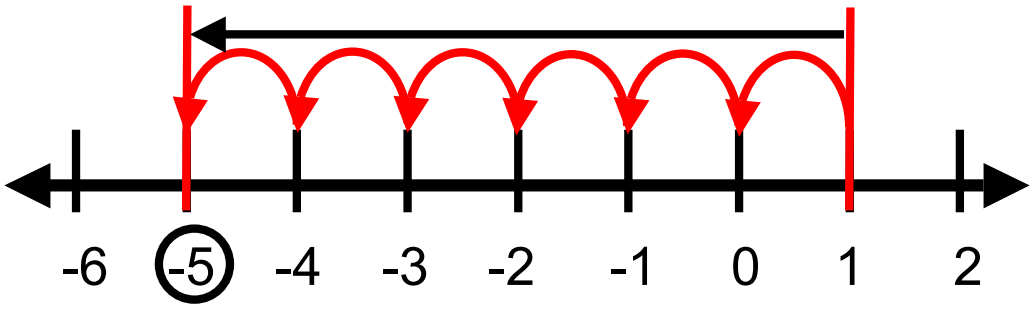
Addition

-5 + 6 = 1



Subtraction

1 – 6 = -5

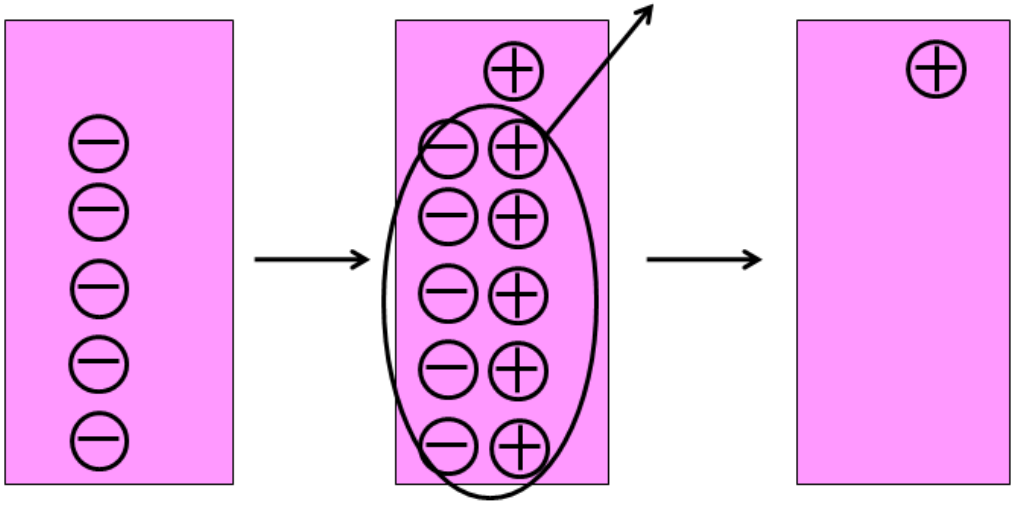


Integer Operations

Addition

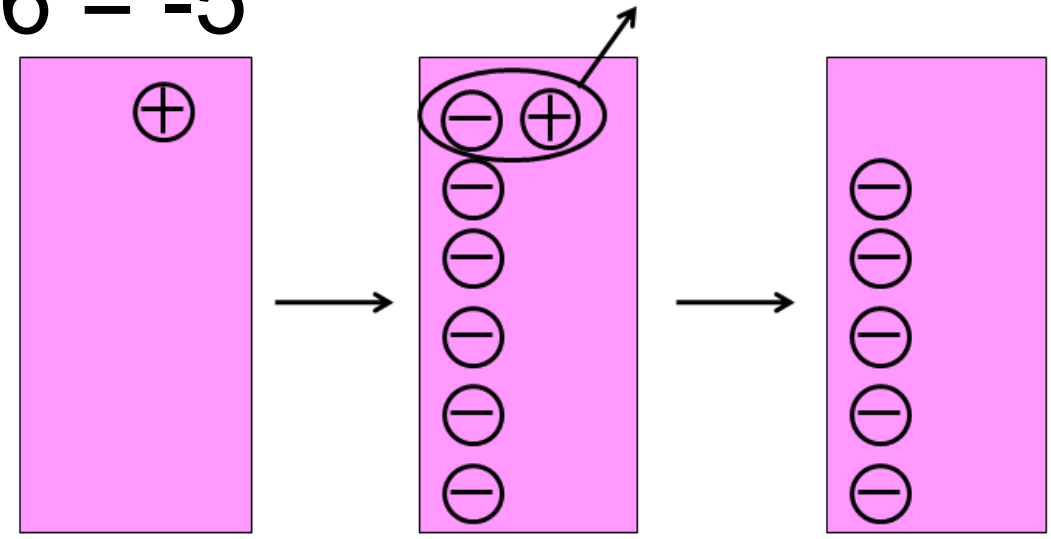
-5 + 6 = 1

⊕ = 1 ⊖ = -1



Subtraction

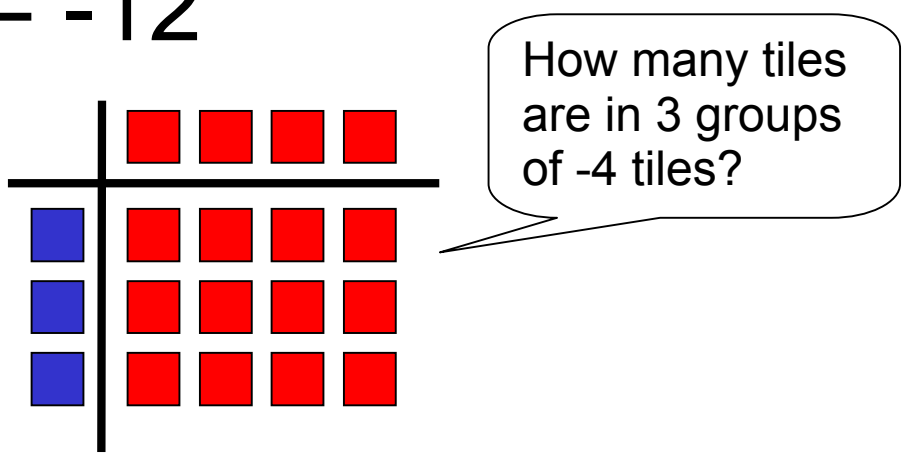
1 – 6 = -5



Integer Operations

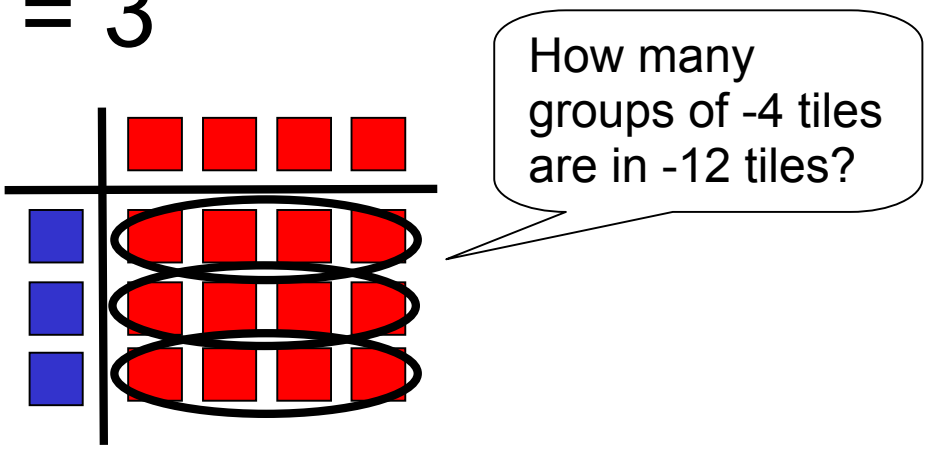
Multiplication

$3 \bullet (-4) = -12$

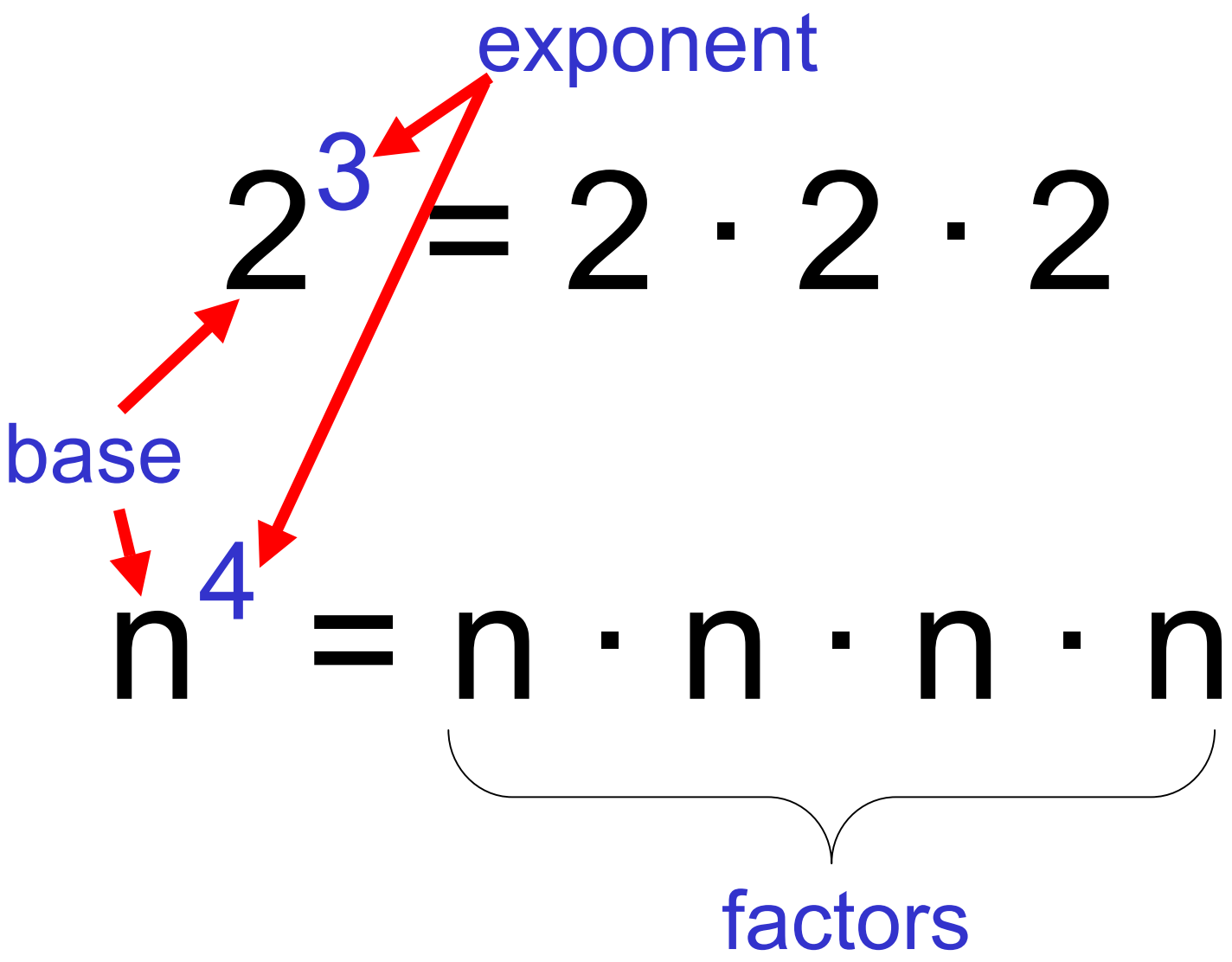


Division

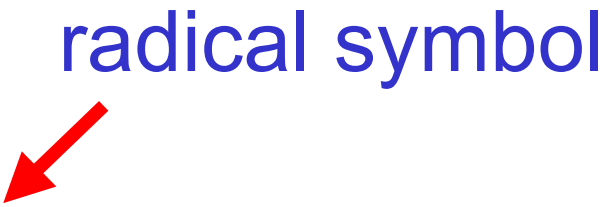
$-12 \div -4 = 3$



Exponential Form



Square Root



$$\sqrt{36} = 6$$

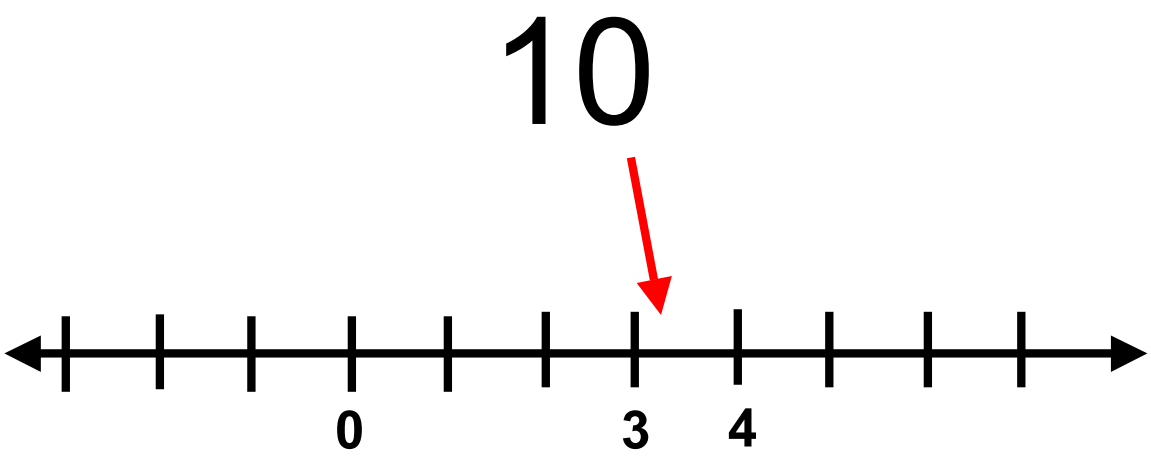
$$36 = 6 \cdot 6 = 62 = 6$$

Squaring a number and taking a square root are inverse operations.

$$-\sqrt{36} = -6$$

$$(-6)^2 = -6 \cdot -6 = 36$$

Square Root



between 9 and 16

Perfect Squares

$$0^2 = 0 \cdot 0 = \mathbf{0}$$
$$1^2 = 1 \cdot 1 = \mathbf{1}$$
$$2^2 = 2 \cdot 2 = \mathbf{4}$$
$$3^2 = 3 \cdot 3 = \mathbf{9}$$
$$4^2 = 4 \cdot 4 = \mathbf{16}$$
$$5^2 = 5 \cdot 5 = \mathbf{25}$$

$$\sqrt{\mathbf{16}} = \sqrt{\mathbf{4 \cdot 4}} = \mathbf{4}$$

perfect square

Powers of Ten

	Meaning	**Value**
10^4	$10 \cdot 10 \cdot 10 \cdot 10$	10,000
10^3	$10 \cdot 10 \cdot 10$	1000
10^2	$10 \cdot 10$	100
10^1	10	10
10^0	1	1
10^{-1}	110	0.1
10^{-2}	$110 \cdot 110$	$1100 = 0.01$
10^{-3}	$110 \cdot 110 \cdot 110$	$11000 = 0.001$
10^{-4}	$110 \cdot 110 \cdot 110 \cdot 110$	$110{,}000 = 0.0001$

Scientific Notation

$$a \times 10^{n}$$

a = number greater than or equal to 1 and less than 10

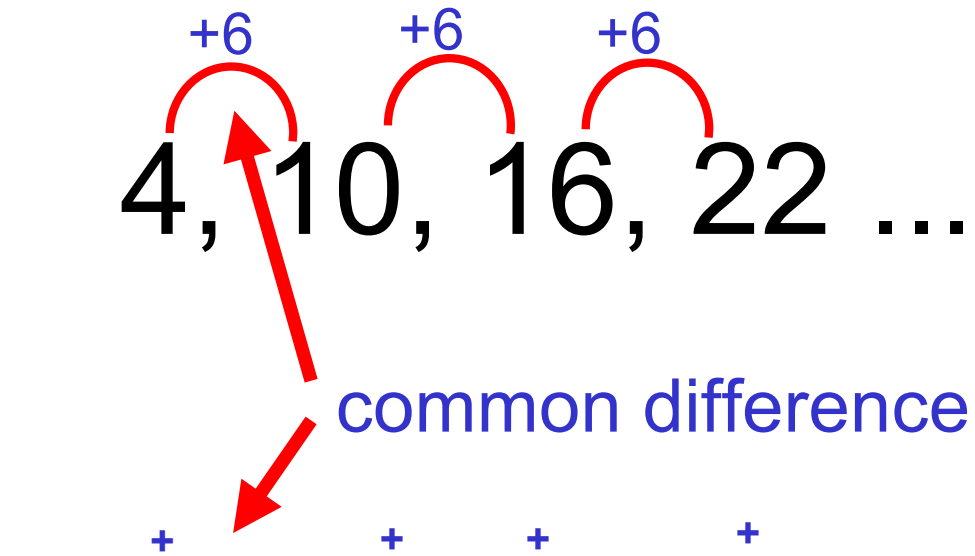
n = integer

$$17{,}500{,}000 = 1.75 \times 10^{7}$$

$$0.0000026 = 2.6 \times 10^{-6}$$

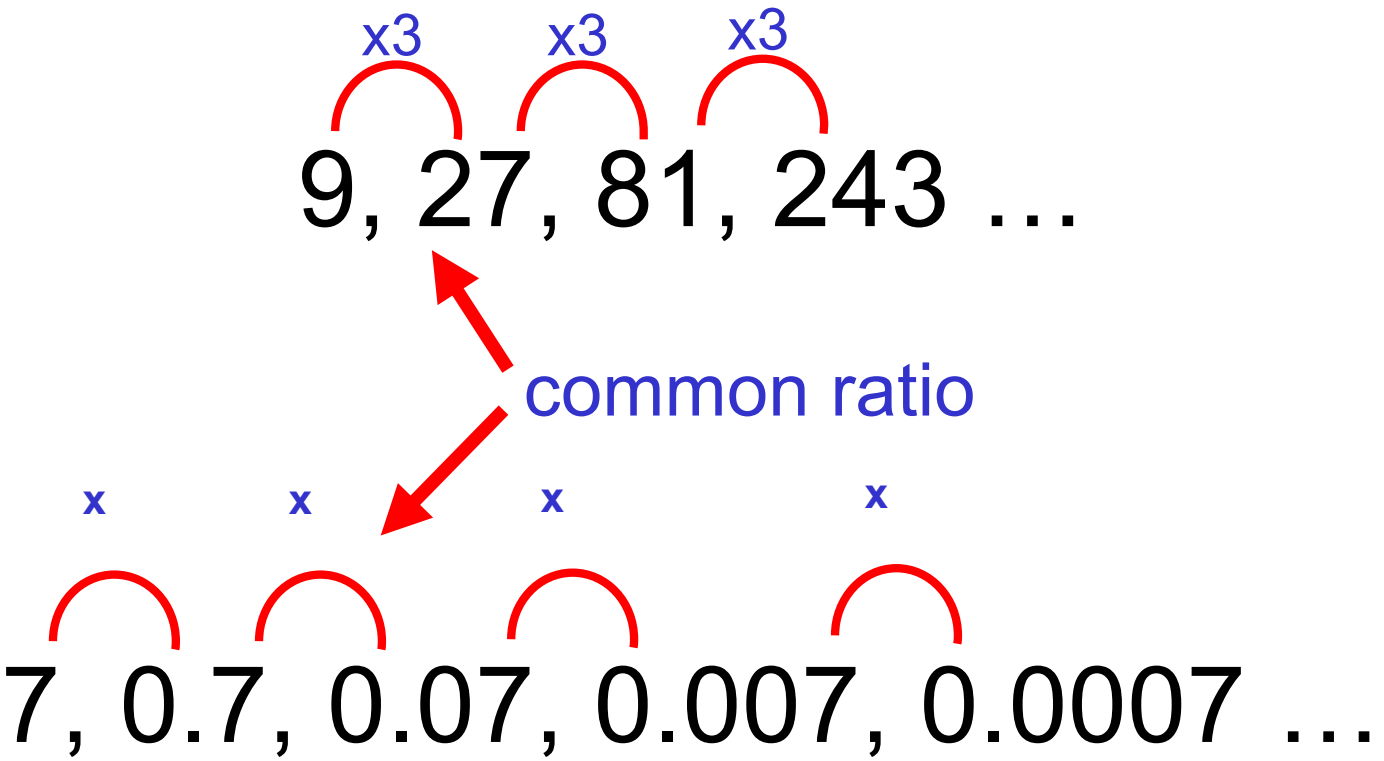
Arithmetic Sequences

What is the next term?



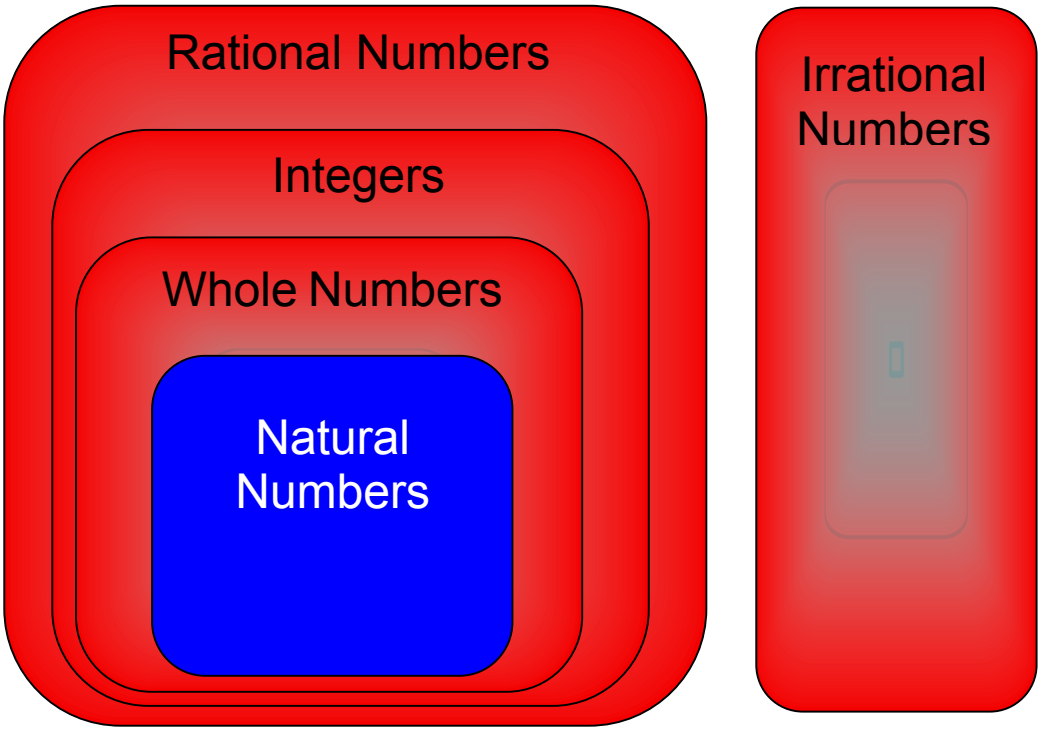
Geometric Sequences

What is the next term?



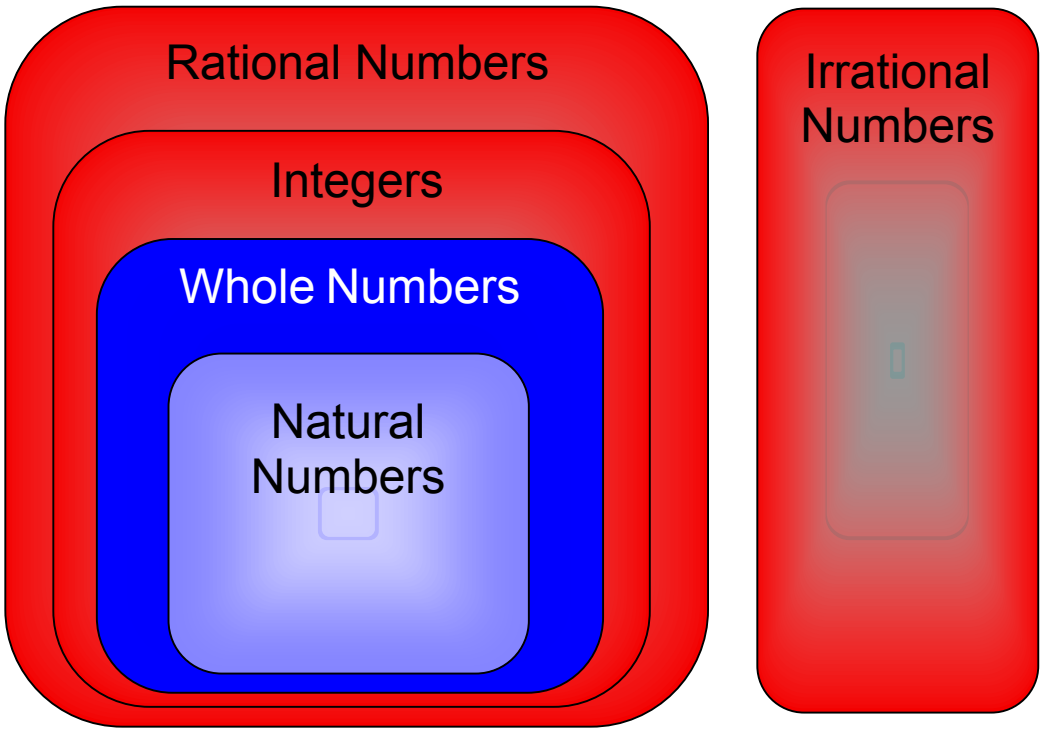
Natural Numbers

The set of numbers
1, 2, 3, 4…



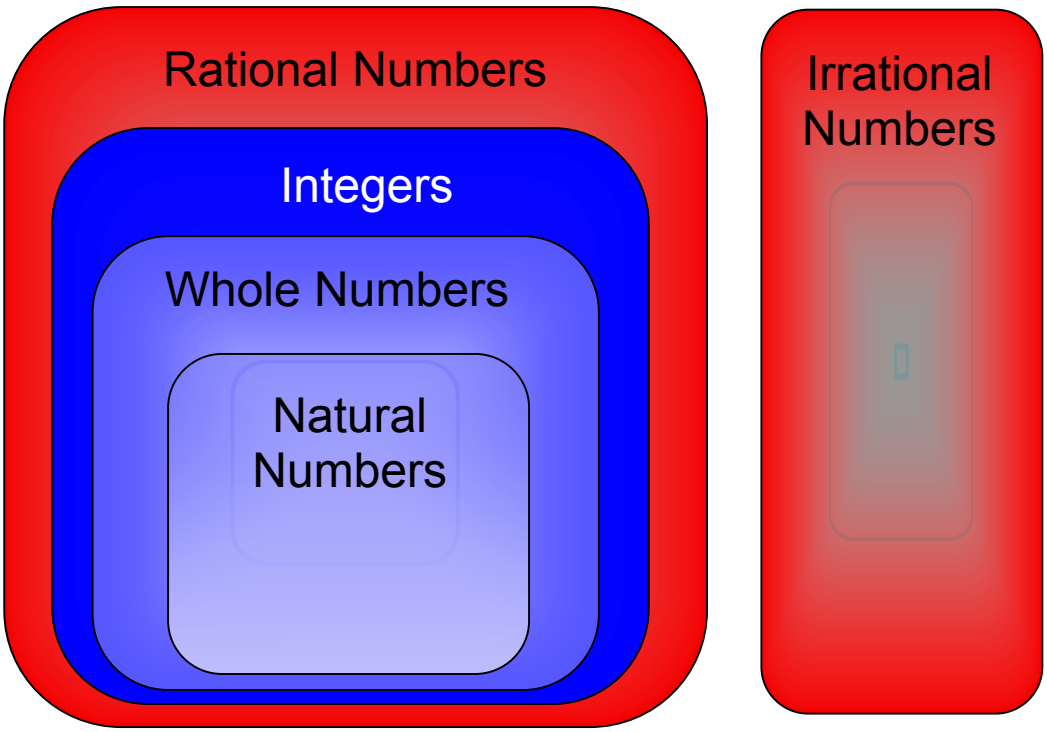
Whole Numbers

The set of numbers 0, 1, 2, 3, 4…

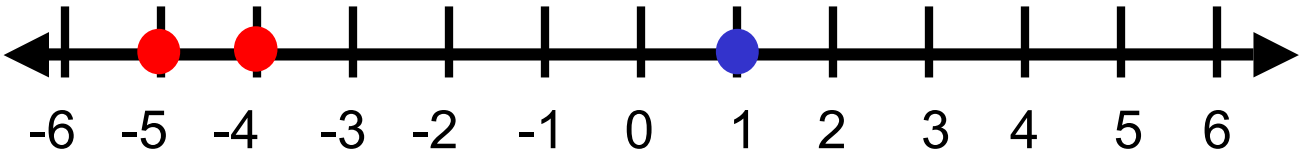


Integers

The set of numbers
…-3, -2, -1, 0, 1, 2, 3…



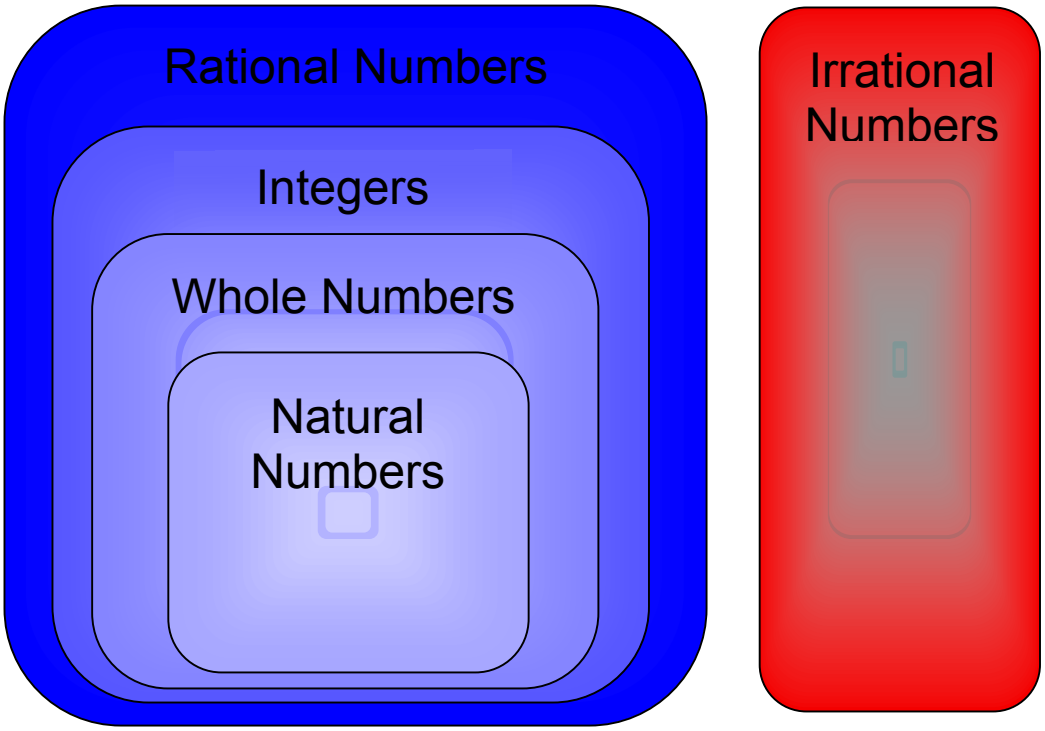
Comparing Integers



-5 < 1 or 1 > -5

-4 > -5 or -5 < -4

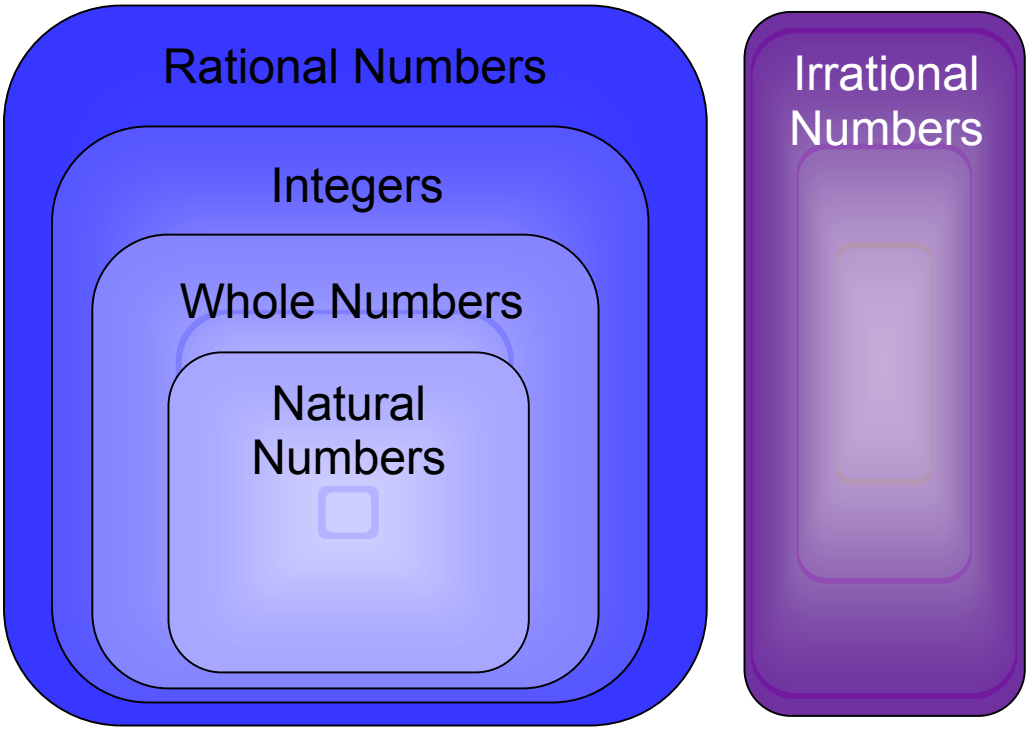
Rational Numbers



A number that can be written as the quotient of two integers

235 -5 0.3 16 137

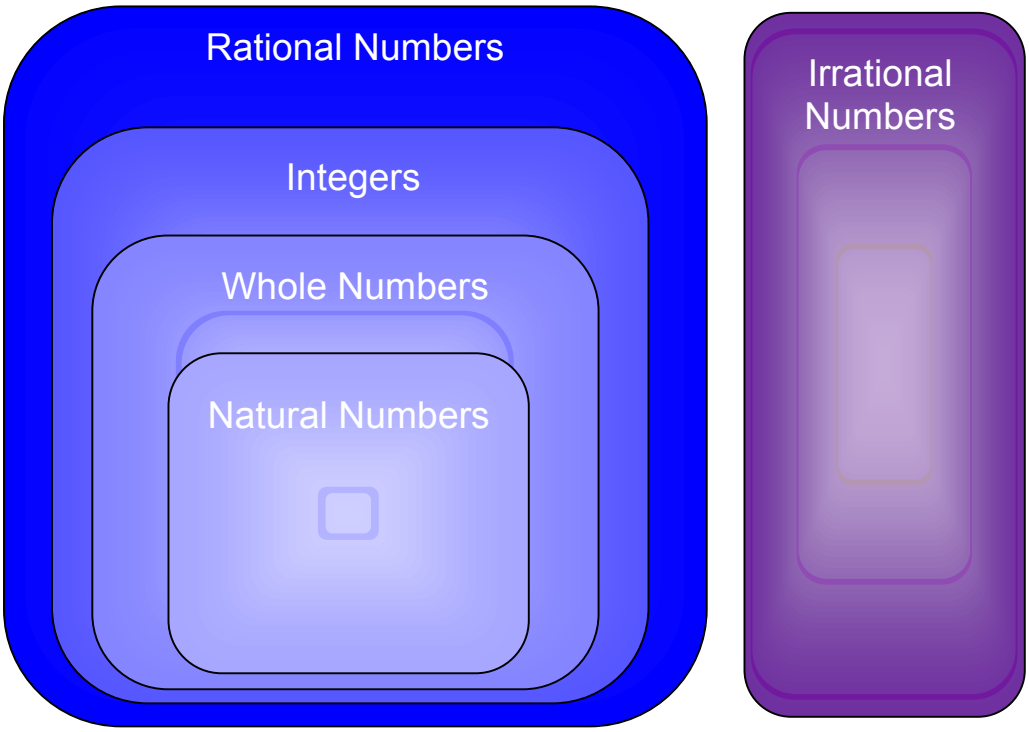
Irrational Numbers



A number that cannot be expressed as the quotient of two integers

7 ▯ -0.2322322322223…

Real Numbers



The set of all rational and irrational numbers

Order of Operations

Grouping Symbols
- ( )
- { }
- [ ]
- |abs|
- Fraction bar

Exponents

Multiplication
Division

Left to right

Addition
Subtraction

Left to right

Proportion

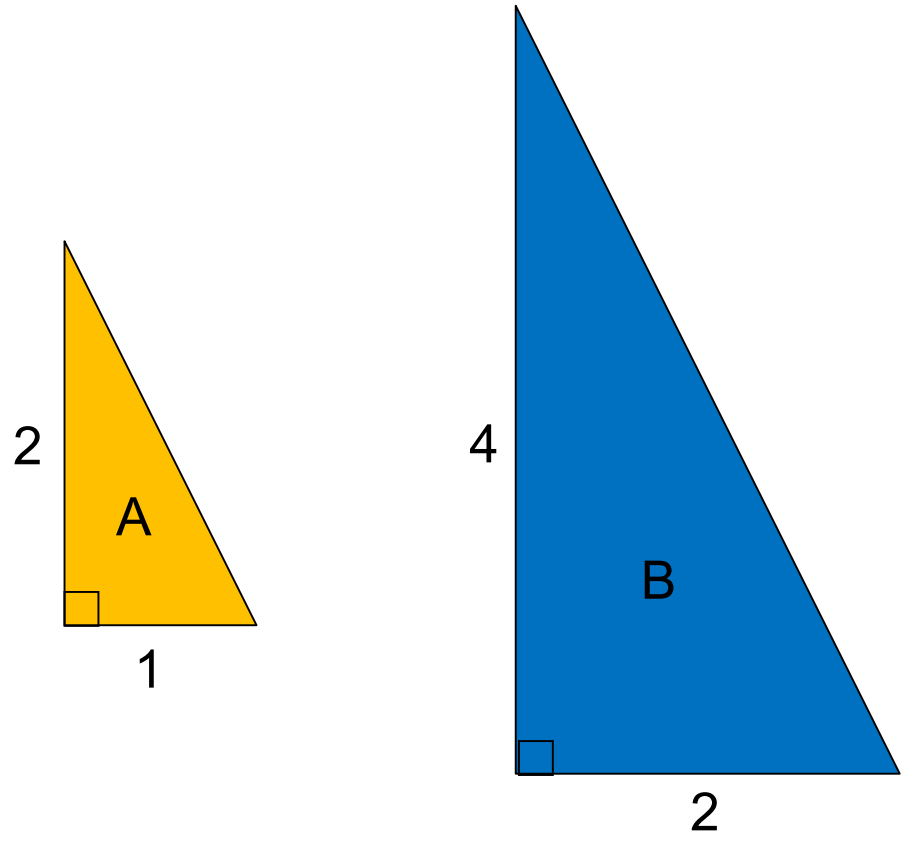
$$\frac{a}{b} = \frac{c}{d}$$

$a:b = c:d$

a is to b as c is to d

Scale Factor

Figures A and B are similar.



What is the scale factor from A to B?

Scale factor = 2

What is the scale factor from B to A?

Scale factor = 1/2

Percent

Per hundred

$56\% = 56/100 = 14/25 = 0.56$

Unit Rate

$4 per gallon = $41 gallon

70 miles per hour = 70 miles1 hour

Percent of Increase

Percent of change = $\frac{\text{new} - \text{original}}{\text{original}}$

Was $3.25 per gallon

Now $3.85 per gallon

What is the percent of increase?

$$\frac{3.85 - 3.25}{3.25}$$

increase of 18%

Percent of Decrease

Percent of change = $\frac{\text{new} - \text{original}}{\text{original}}$

Was $1200
Now only $900

What is the percent of decrease?

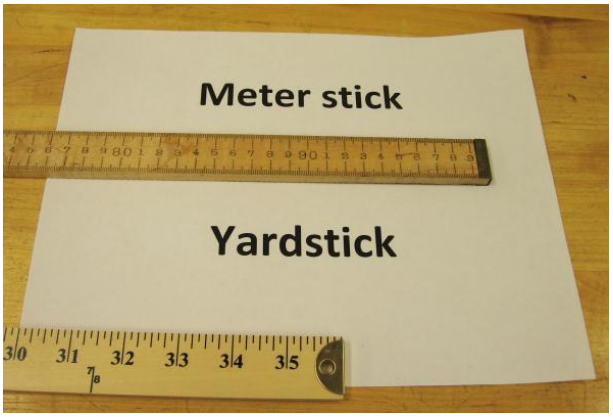
$$\frac{900 - 1200}{1200}$$

decrease of 25%

Ballpark Comparisons

Length

1 inch or
2.5 centimeter



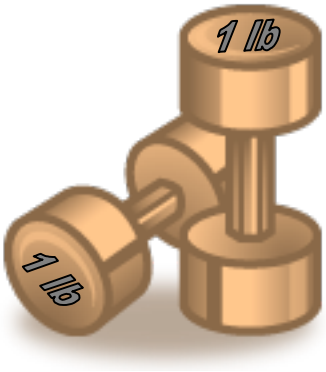
1 yard < 1 meter

Ballpark Comparisons

Weight/Mass

≈

~



≈

Ballpark Comparisons
Volume

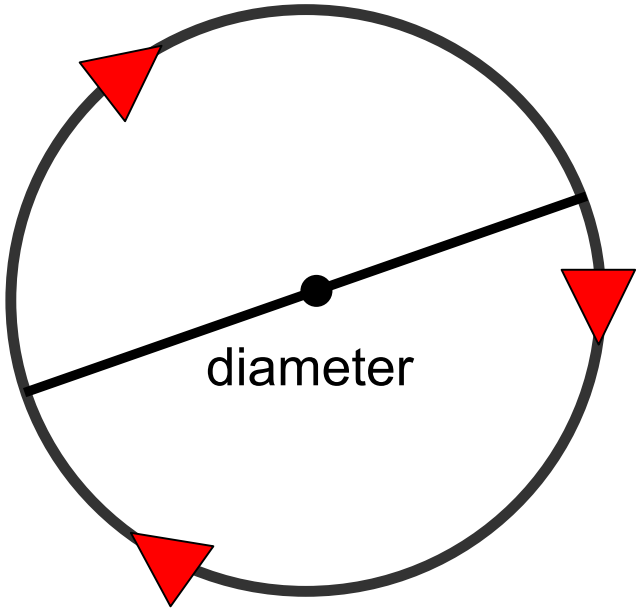
Ballpark Comparisons

Temperature

	Fahrenheit	Celsius
Water freezes	32°F	0°C
Water boils	212°F	100°C
Body Temperature	98°F	37°C
Room Temperature	70°F	20°C

Pi

$\pi \approx 3.14159\ldots$

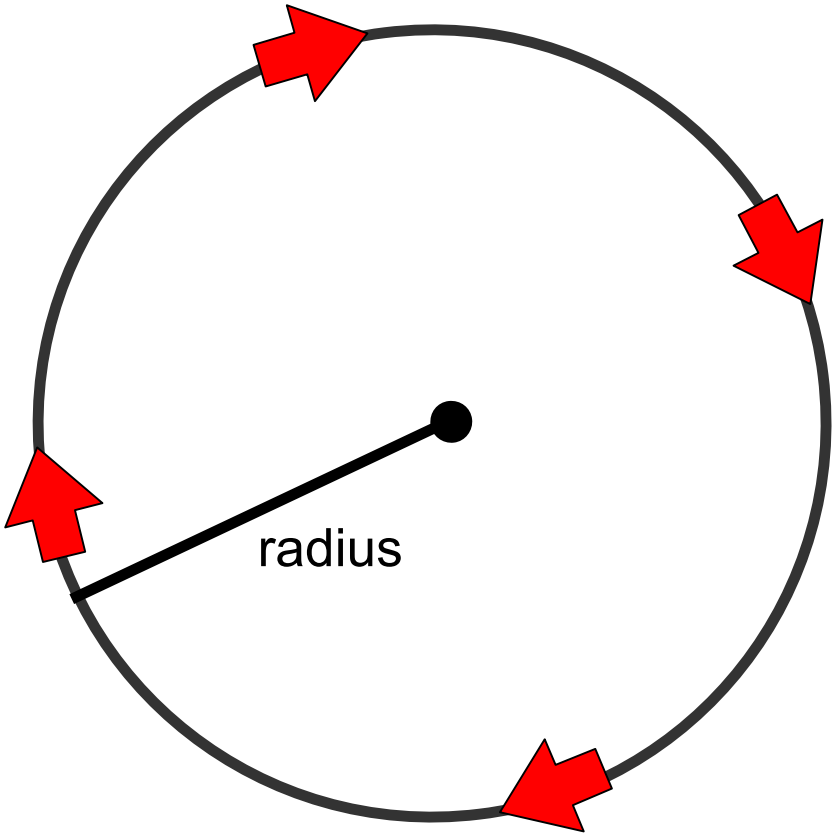


$\pi =$

circumferencediamet

er

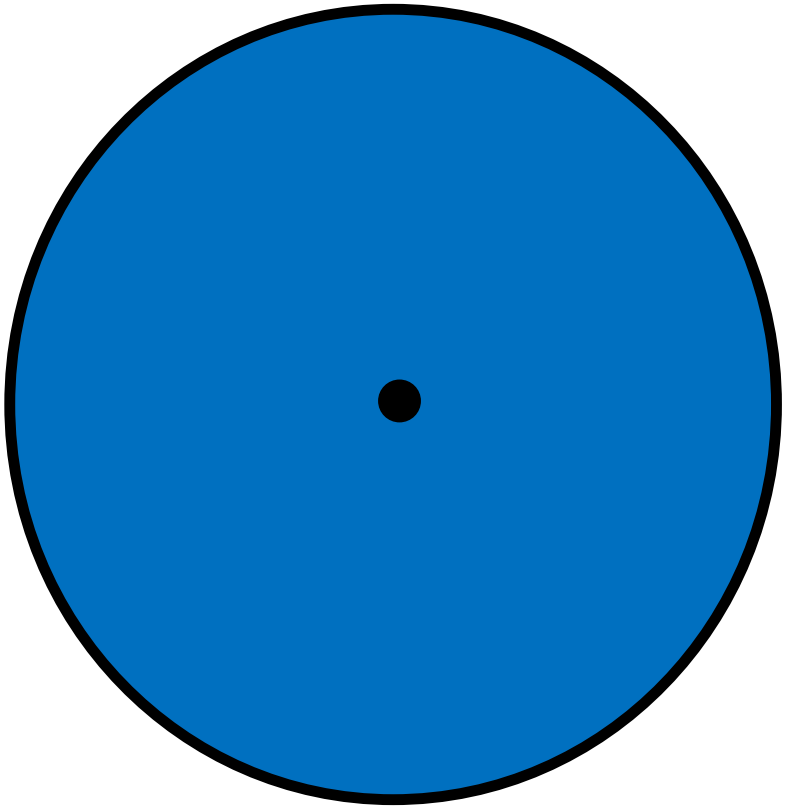
Circumference



$$C = 2\pi r$$

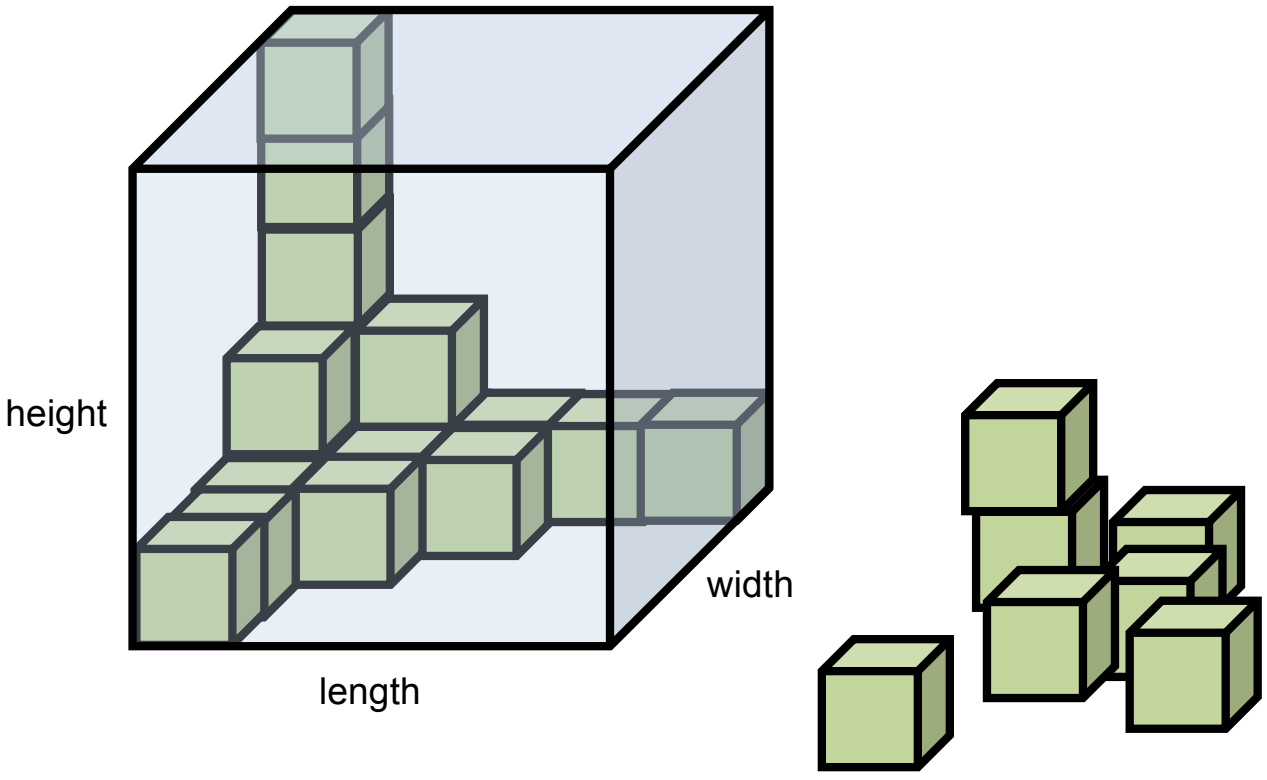
C = perimeter of a circle

Area of a Circle



$$A = \pi r^2$$

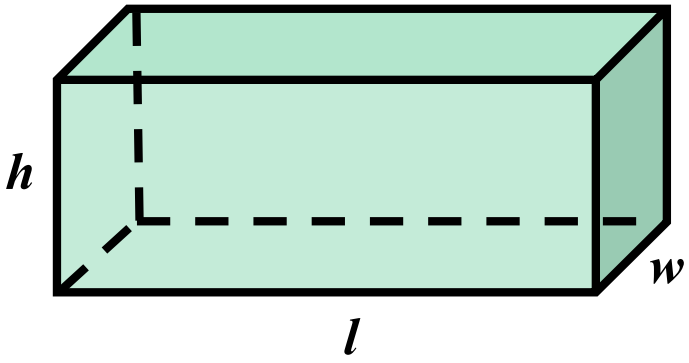
Volume of a Prism

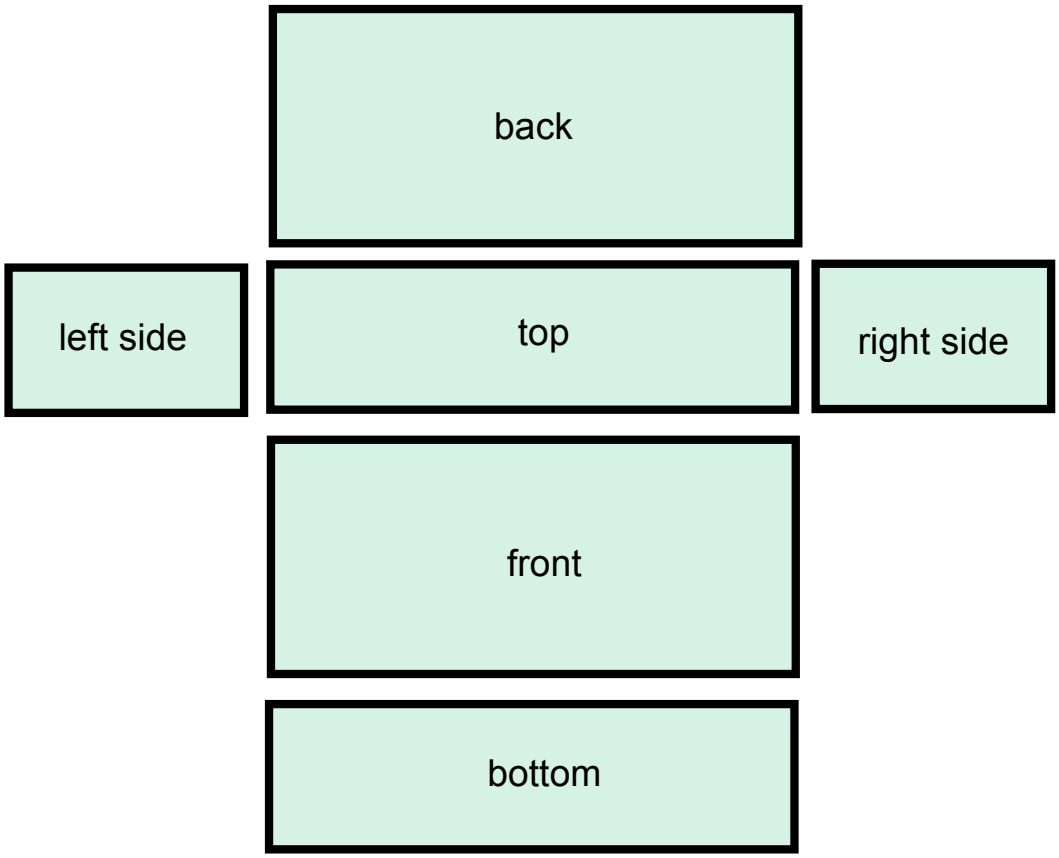


Volume = length x width x height

V = lwh

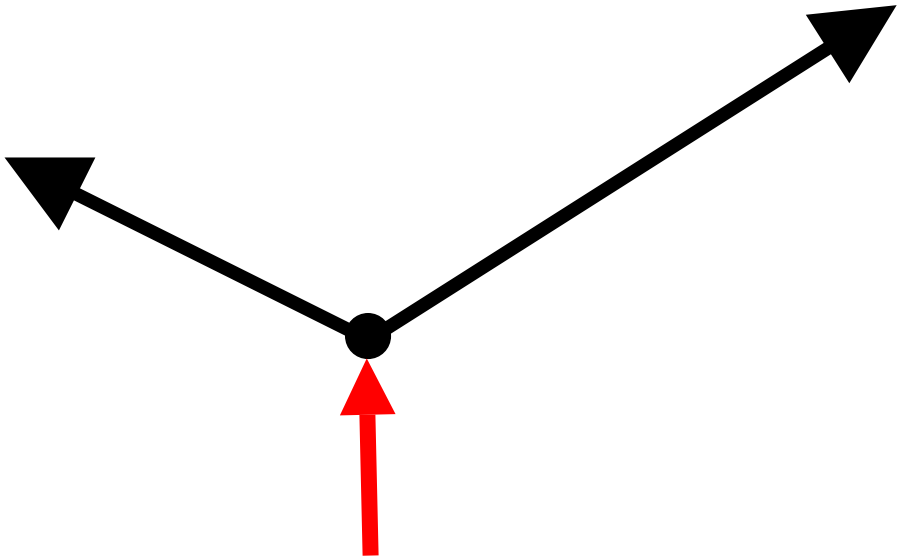
Surface Area





Surface Area (*S.A.*) = sum of areas of faces

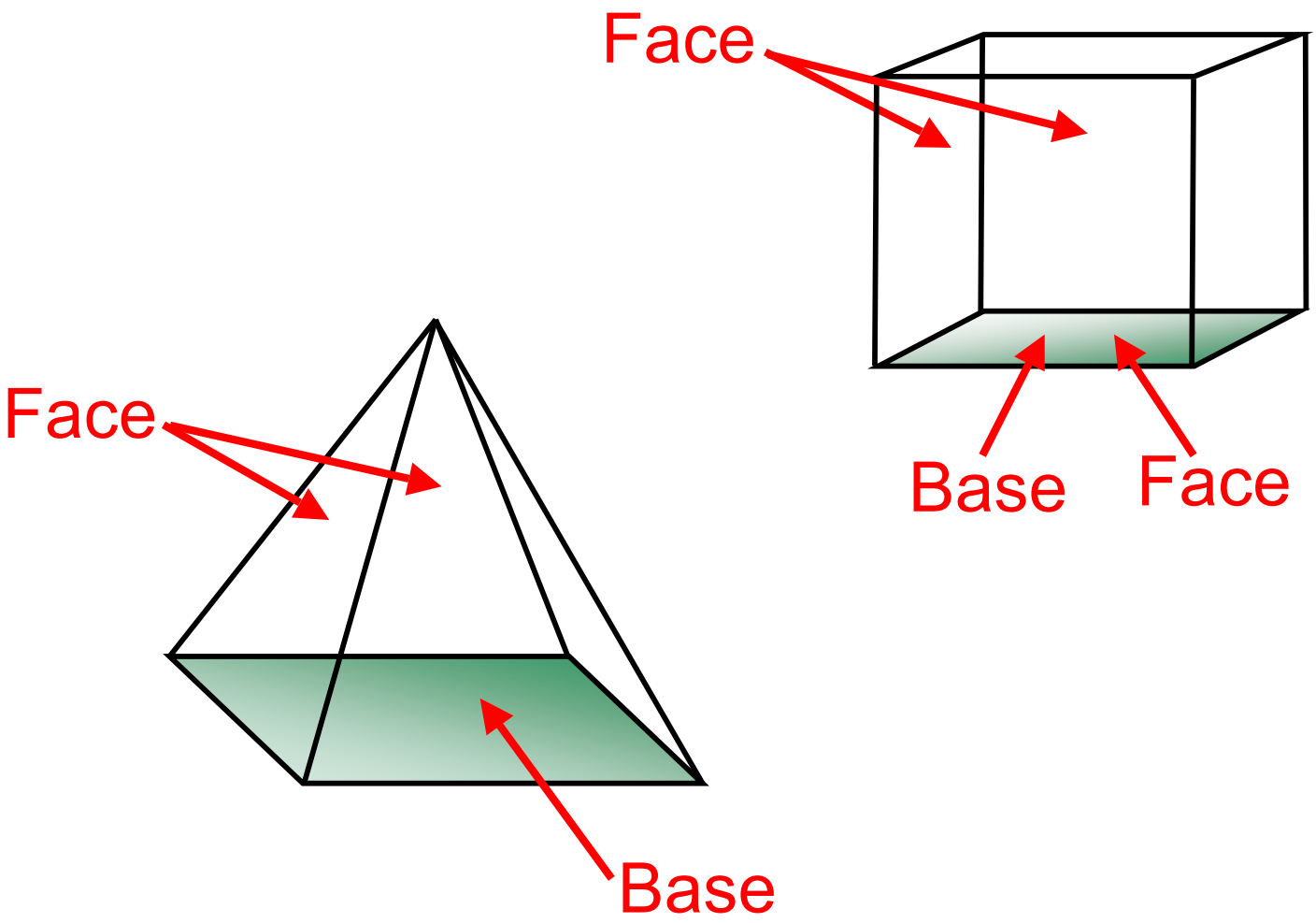
Vertex



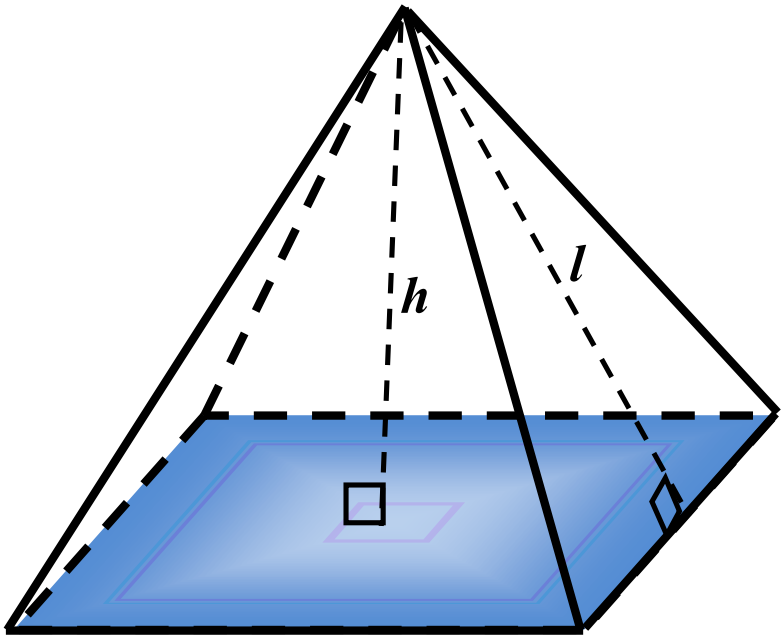
vertex

vertices

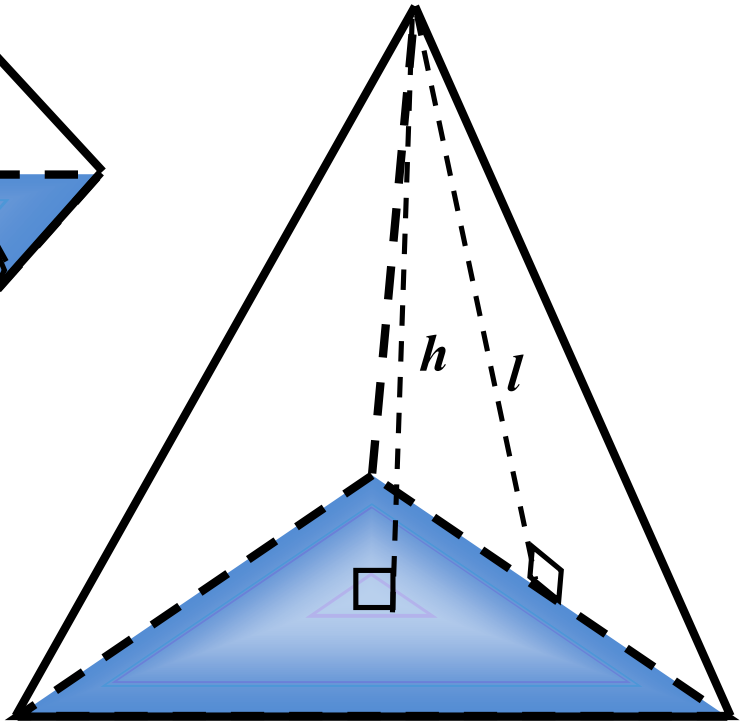
Face and Base



Pyramid



B = area of base
p = perimeter of base

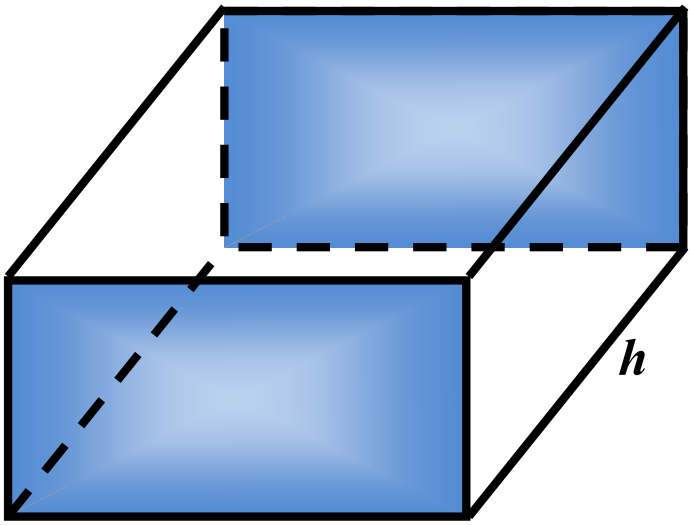


B = area of base
p = perimeter of base

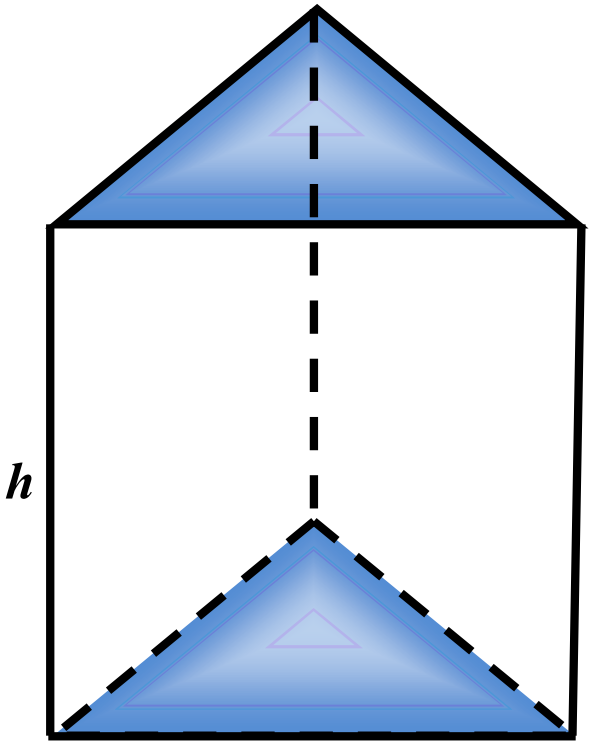
$V = Bh$

$S.A. = lp + B$

Prism



B = area of base
p = perimeter of base

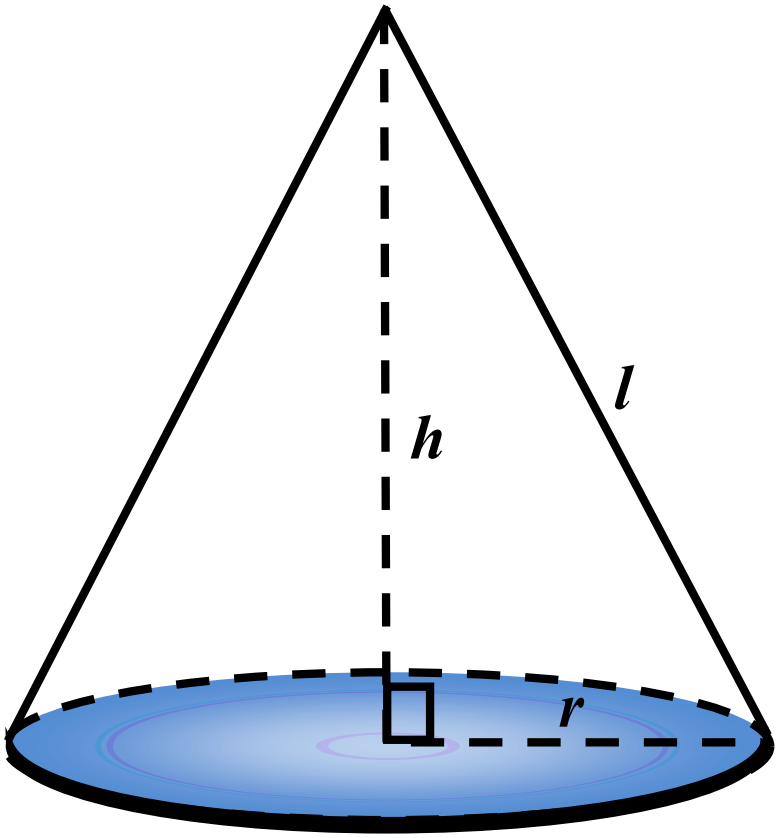


B = area of base
p = perimeter of base

$$V = Bh$$

$$S.A. = hp + 2B$$

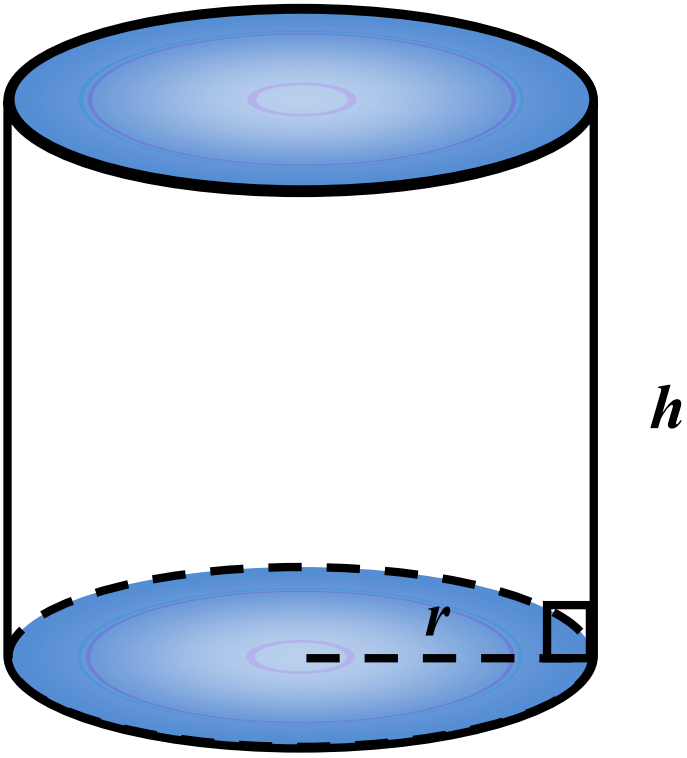
Cone



$$V = \pi r^2 h$$

$$S.A. = \pi r^2 + \pi r l$$

Cylinder

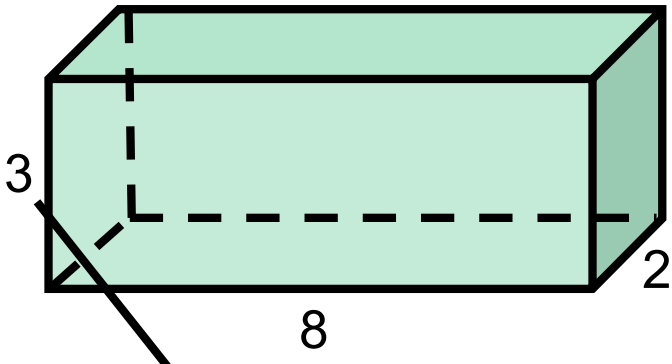


$$V = \pi r^2 h$$

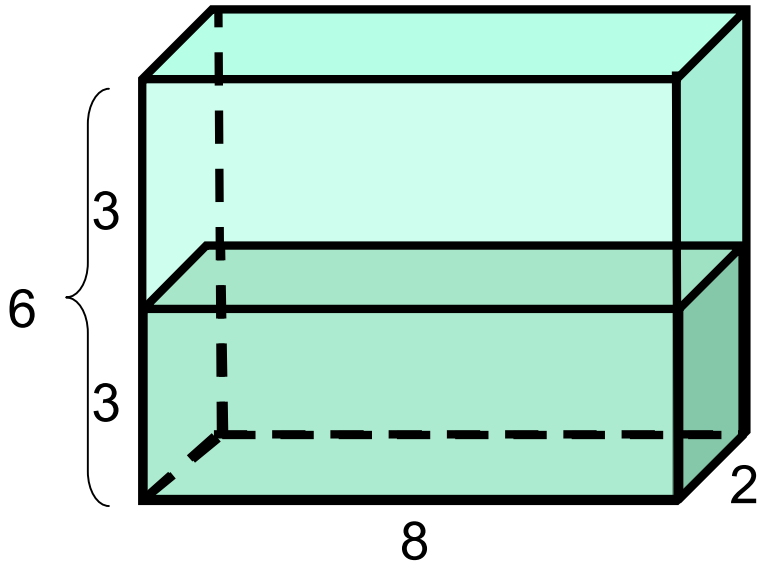
$$S.A. = 2\pi r^2 + 2\pi r h$$

Volume

Changing one attribute

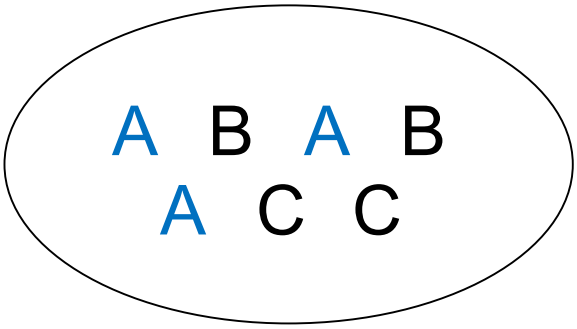


Height increases to 6

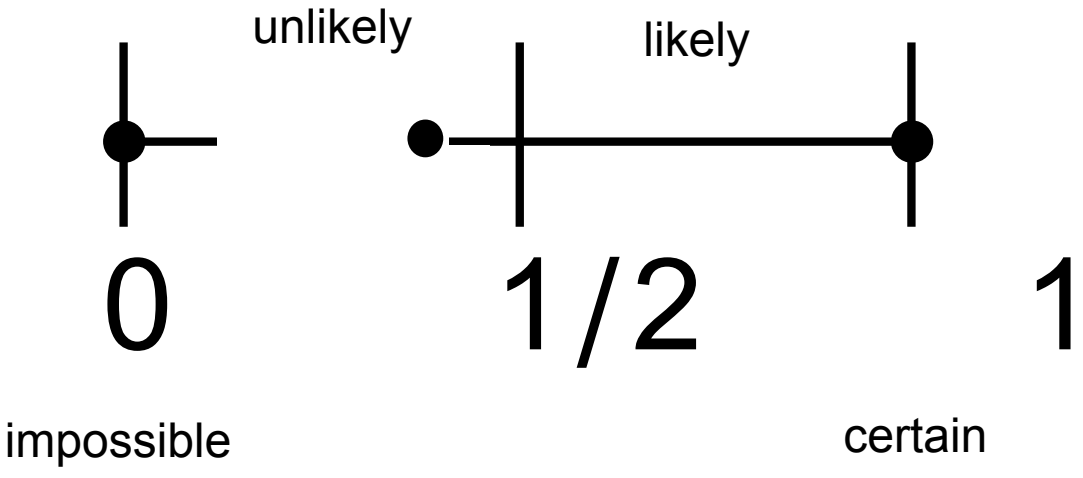


What happens to the volume?

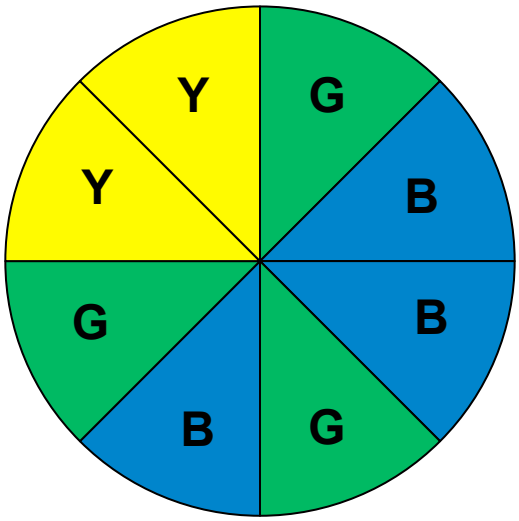
Probability



$$P(A) = 3/7$$



Probability of Independent Events



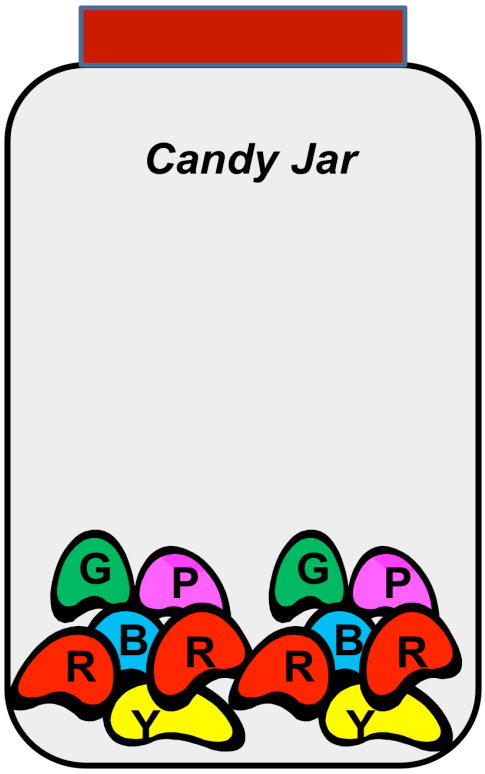
P(green) = 3/8
P(yellow) = 2/8 = 1/4

P(green and yellow) =

P(green) • P(yellow) = 3/8·1/4 = 3/32

Probability of Dependent Events

What is the probability of getting a red jelly bean on first pick and then without replacing it, getting a green jelly bean on the second pick?



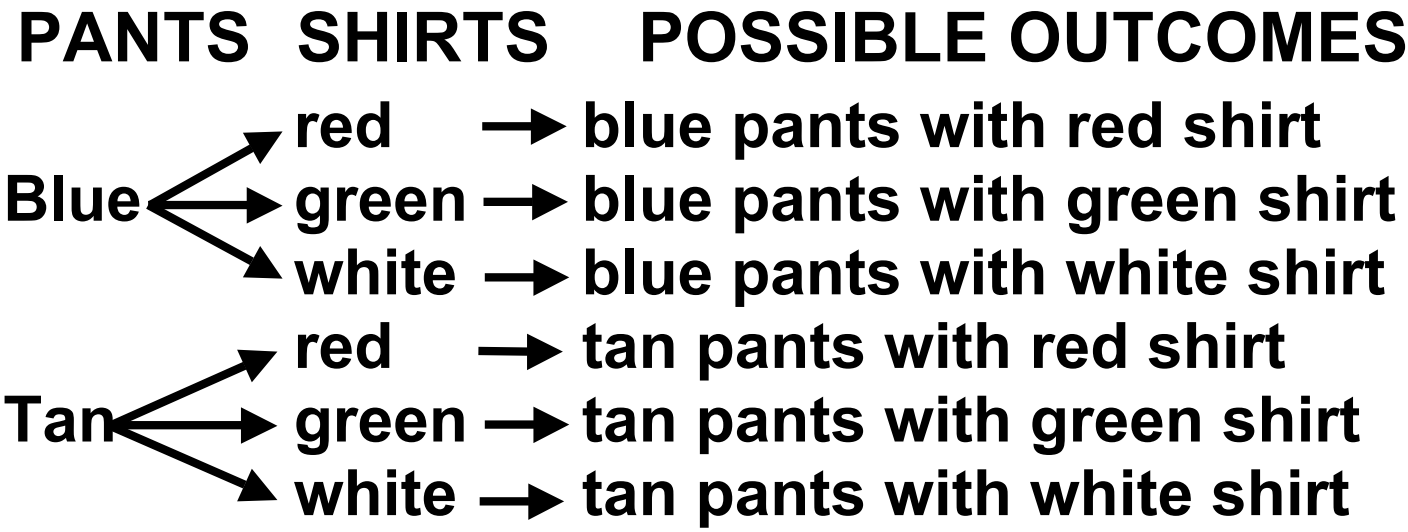
$$P(\text{red}) \cdot P(\text{green after red}) = \frac{4}{12} \cdot \frac{2}{11} = \frac{8}{132} = \frac{2}{33}$$

Fundamental Counting Principle

If there are *m* ways for one event to occur and *n* ways for a second event to occur, then there are $m \cdot n$ ways for both events to occur.

Tree Diagram

Joe has two pairs of pants (blue and tan). He also has three shirts (red, green and white). List the possible outfits that Joe can make.



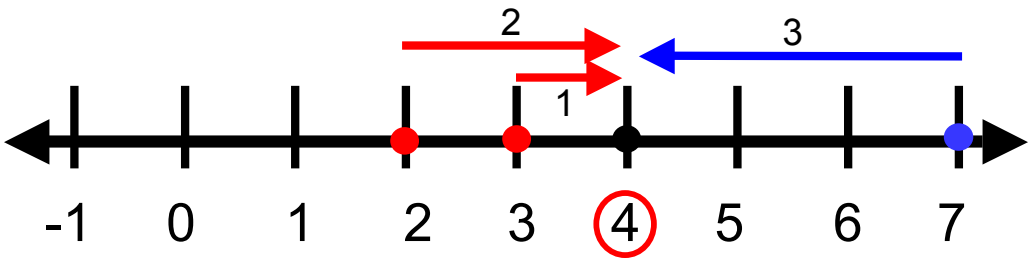
2 • 3 or 6 possible outcomes

MEAN

a measure of central tendency

2, 3, 4, 7

Balance Point



Numerical Average

$$\frac{2+3+4+7}{4} = \frac{16}{4} = 4$$

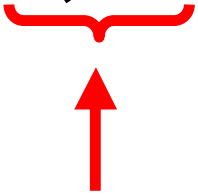
Median

a measure of central tendency

6, 7, 8, 9, 9

8 = median

5, 6, 8, 9, 11, 12

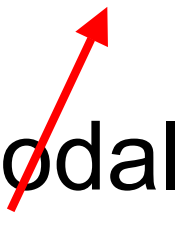


8.5 = median

MODE

a measure of central tendency

Data Sets	Mode
2, 3, 3, 3, 5, 5, 9, 10	3
5.2, 5.4, 5.5, 5.6, 5.8, 5.9, 6.0	none
1, 1, 2, 5, 6, 7, 7, 9, 11, 12	1, 7



Range

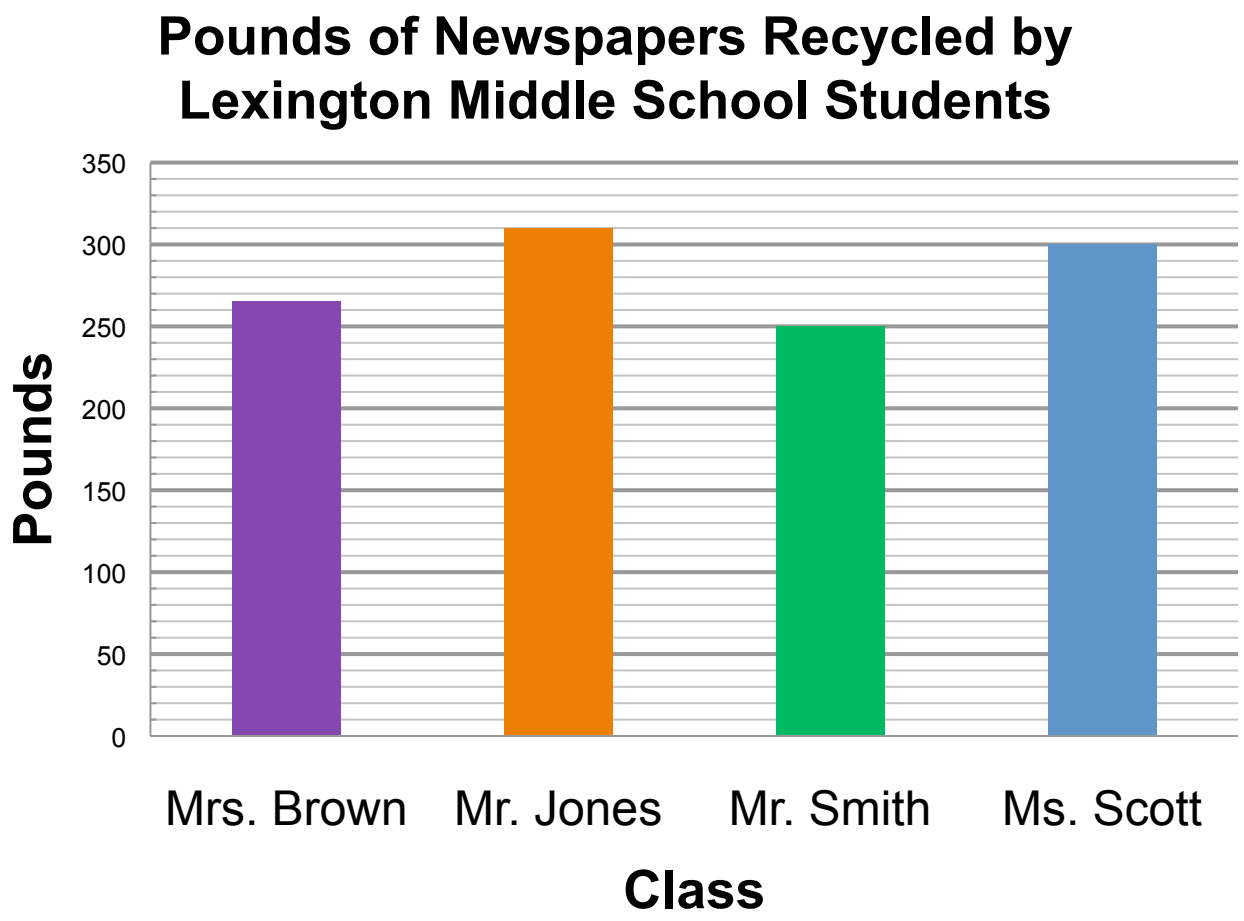
Data set

212, 3, 334, 378, 5, 512, 916, 1045, 1512, 20

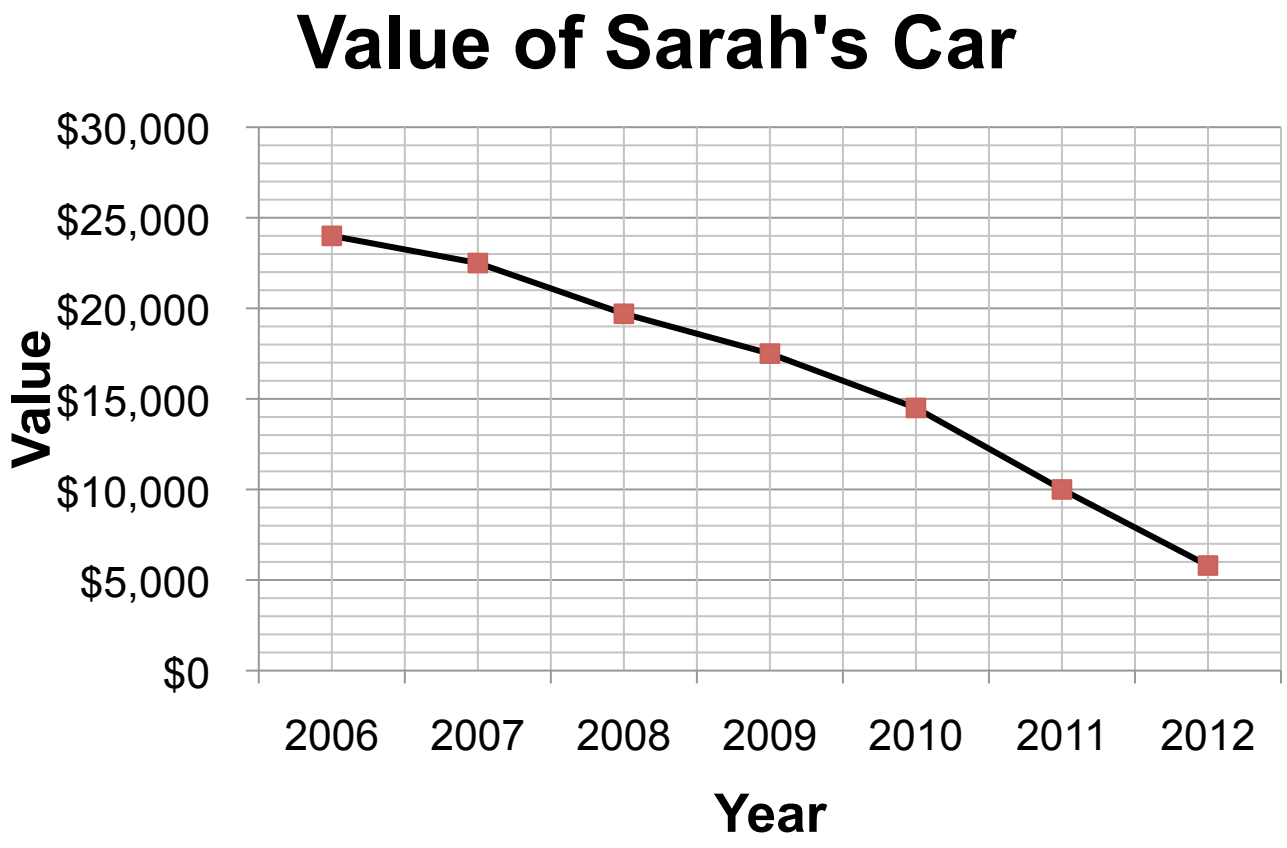
$20 - 2\,12 = 17\,12$

Range = $17\,12$

Bar Graph


Pounds of Newspapers Recycled by Lexington Middle School Students
Pounds
350
300
250
200
150
100
50
0
Mrs. Brown
Mr. Jones
Mr. Smith
Ms. Scott
Class

Line Graph


Value of Sarah's Car
$30,000
$25,000
$20,000
$15,000
$10,000
$5,000
$0
Value
2006
2007
2008
2009
2010
2011
2012
Year

Stem-and-Leaf Plot

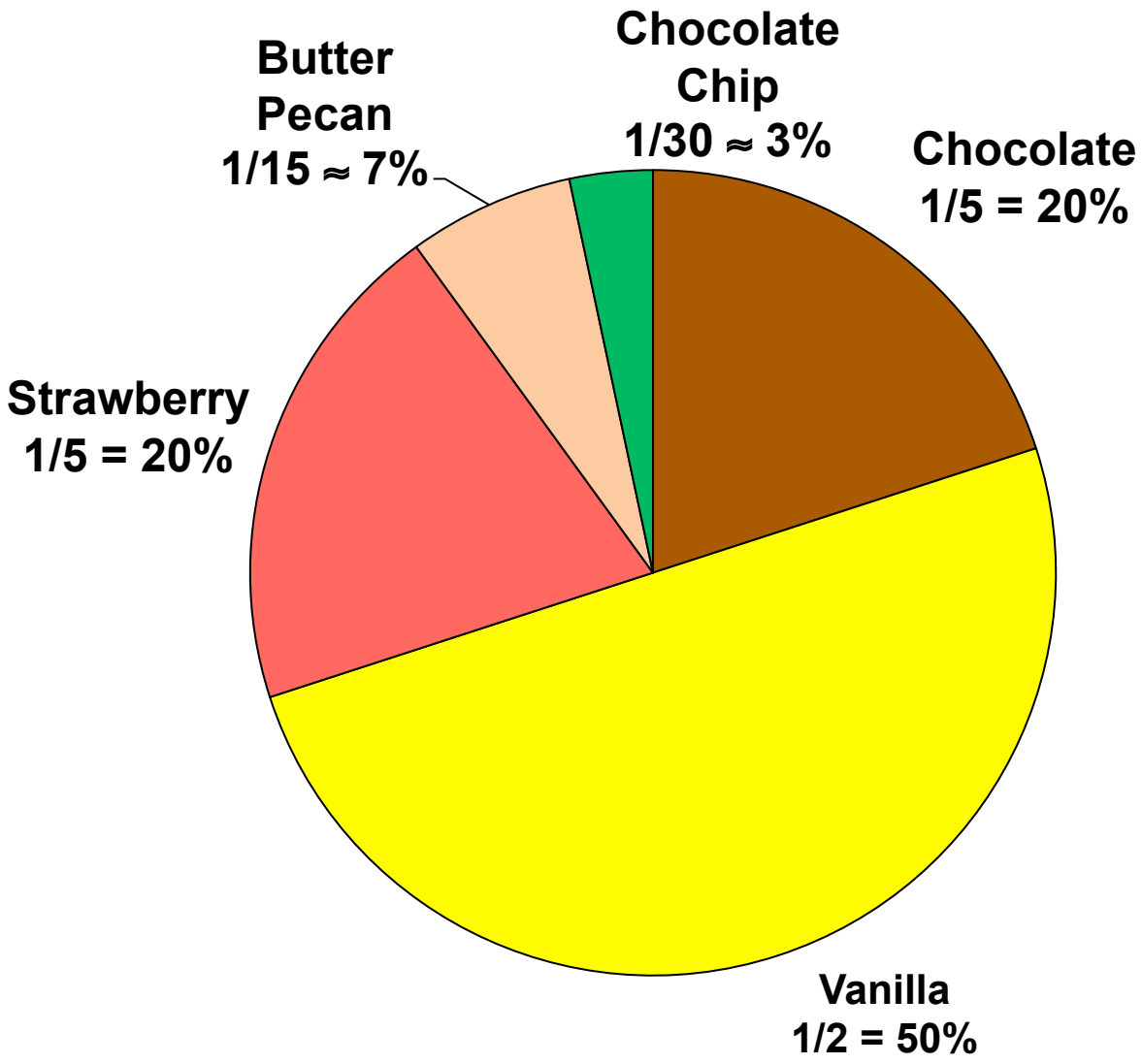
Math Test Scores
56, 65, 98, 82, 64, 71, 78, 86, 95, 91,
59, 70, 80, 92, 76, 82, 85, 91, 92, 73

STEM	LEAF
5	6 9
6	4 5
7	0 1 3 6 8
8	0 2 2 5 6
9	1 1 2 2 5 8

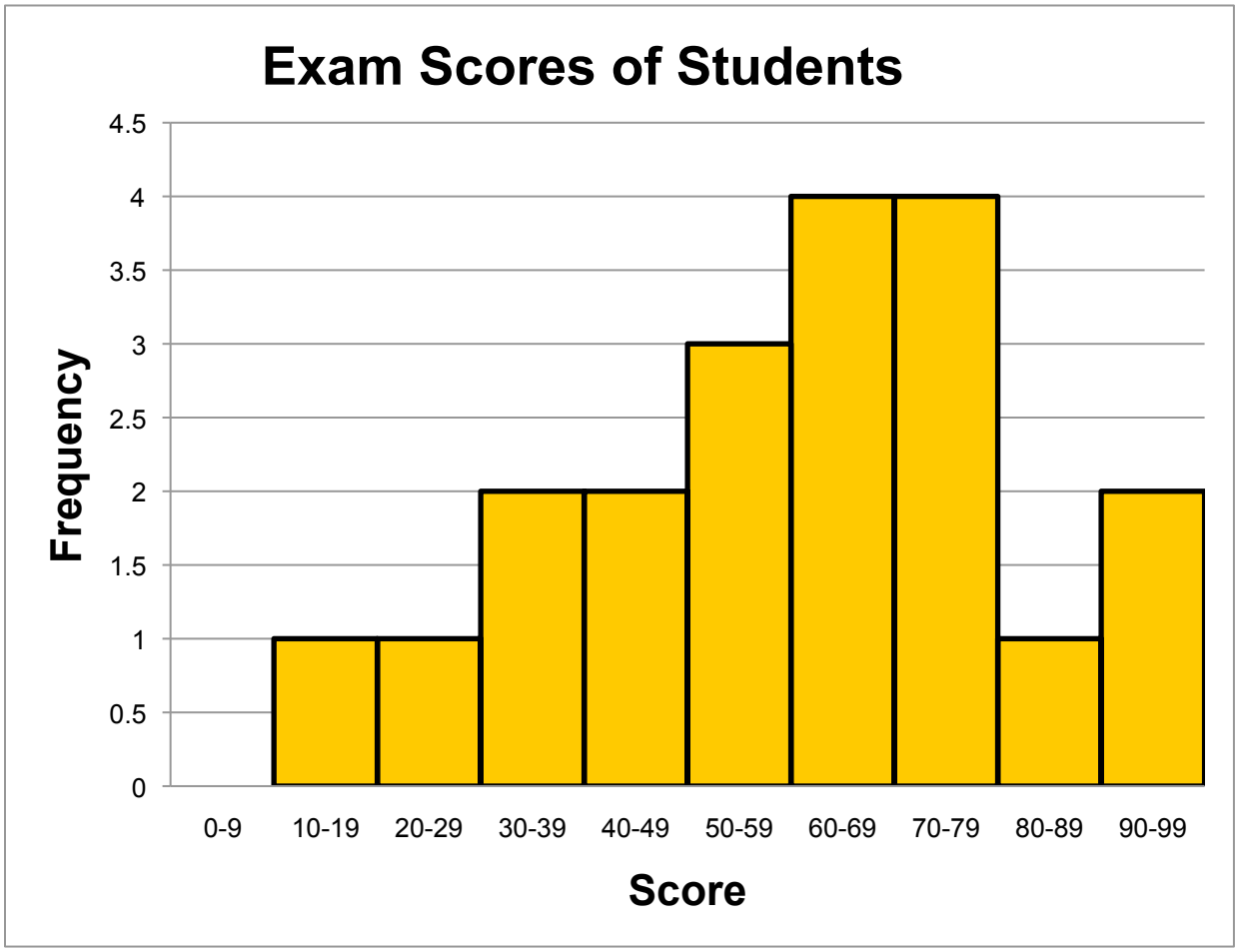
Key: 5|6 means 56

Circle Graph

Favorite Ice Cream

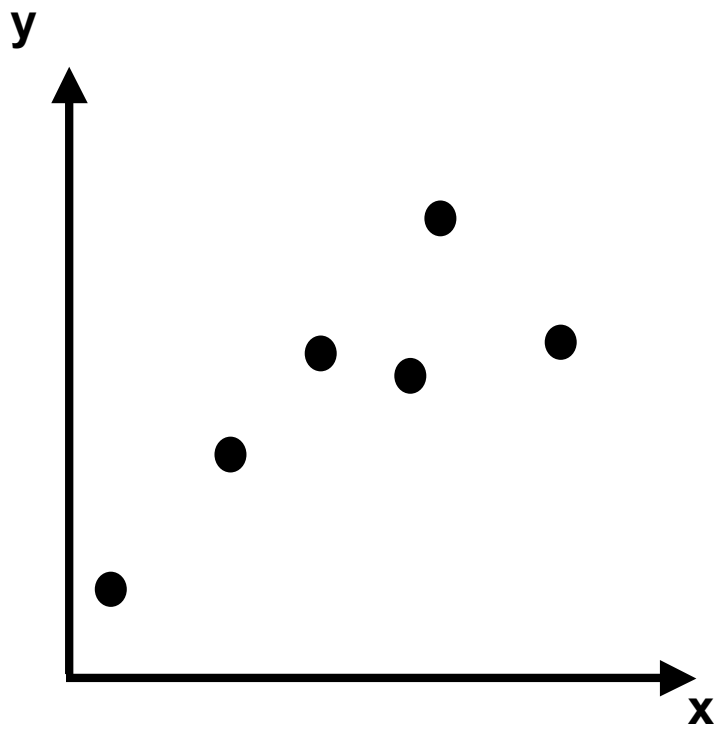

Butter
Pecan
1/15 ≈ 7%
Chocolate
Chip
1/30 ≈ 3%
Chocolate
1/5 = 20%
Strawberry
1/5 = 20%
Vanilla
1/2 = 50%

Histogram


Exam Scores of Students
Frequency
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0-9
10-19
20-29
30-39
40-49
50-59
60-69
70-79
80-89
90-99
Score

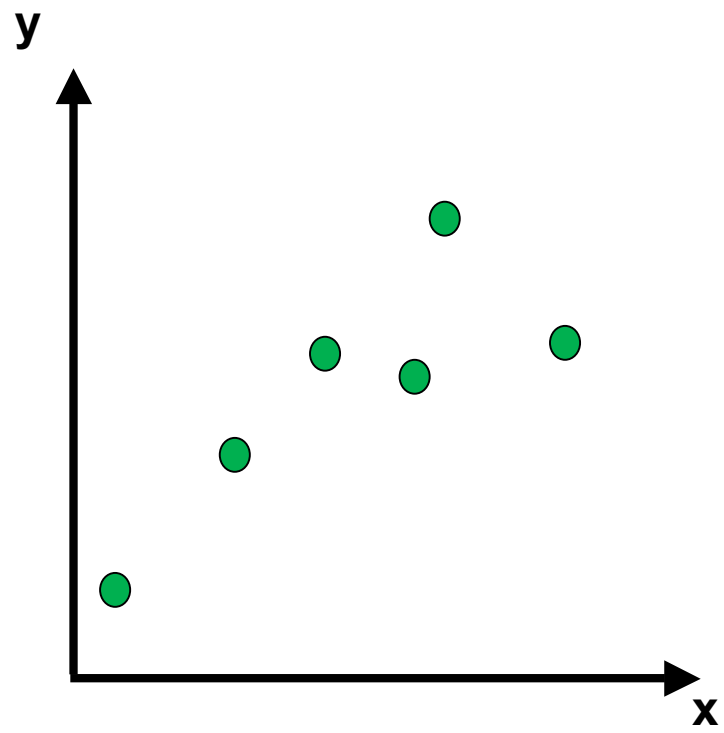
Scatterplot

Illustrates the relationship between two sets of data.



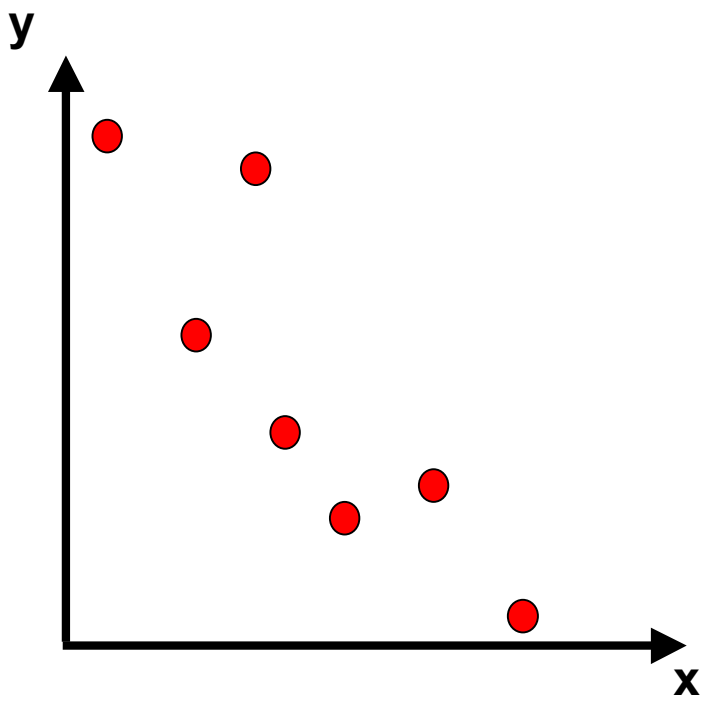
Positive Correlation

y-coordinates increase as x-coordinates increase



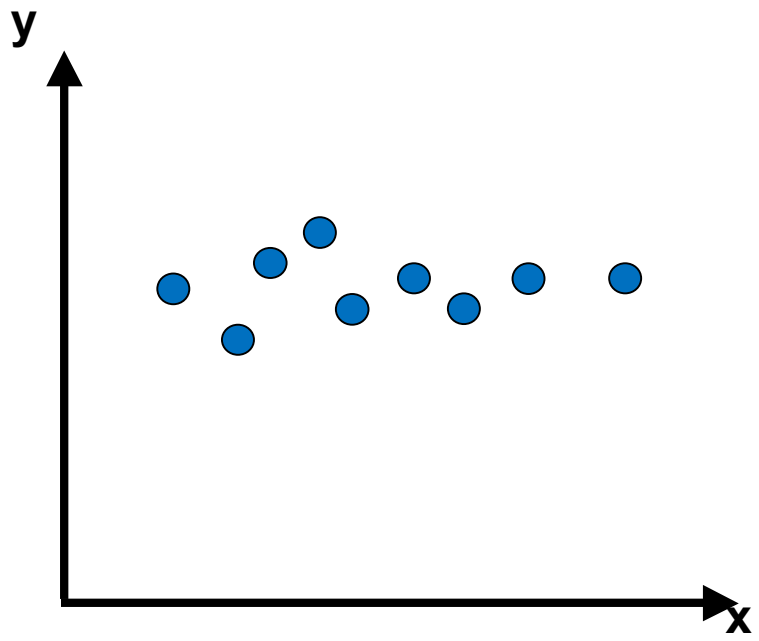
Negative Correlation

y-coordinates decrease as x-coordinates increase



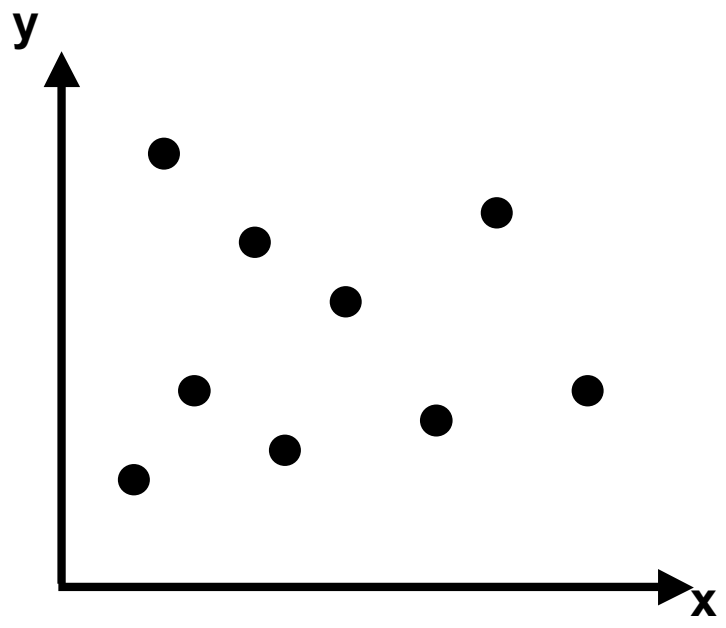
Constant Correlation

y-coordinates remain about the same as x-coordinates increase



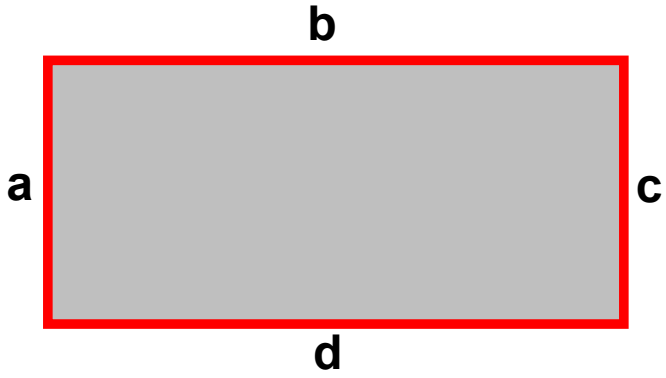
No Correlation

no pattern exists between the x- and y-coordinates

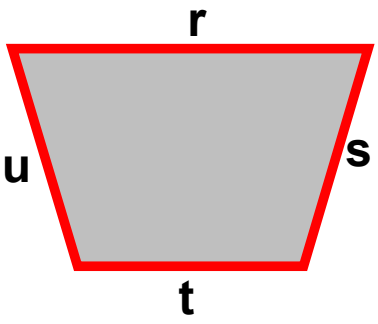


Perimeter

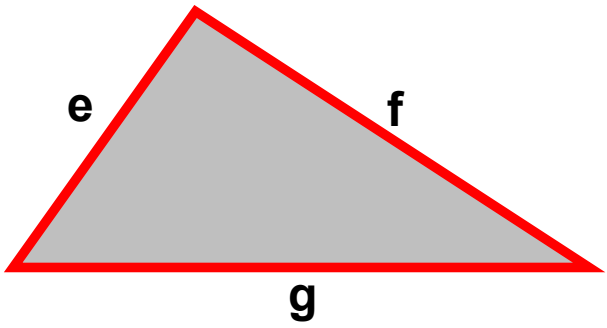
the measure of the distance around a figure



P = a + b + c + d



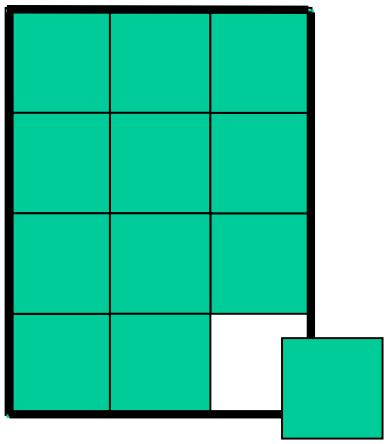
P = r + s + t + u



P = e + f + g

Area

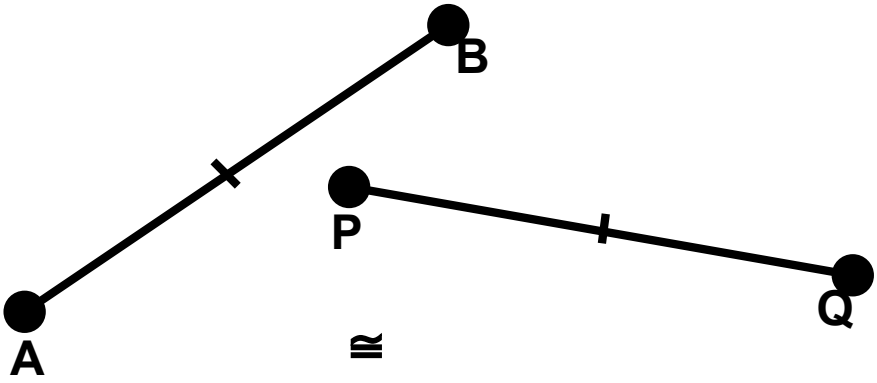
the number of square units needed to cover a surface or figure



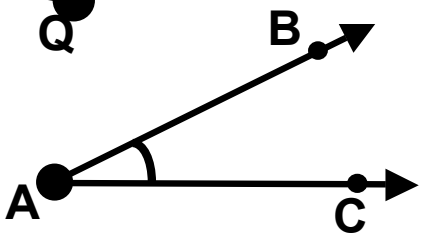
Area = 12 Square Units

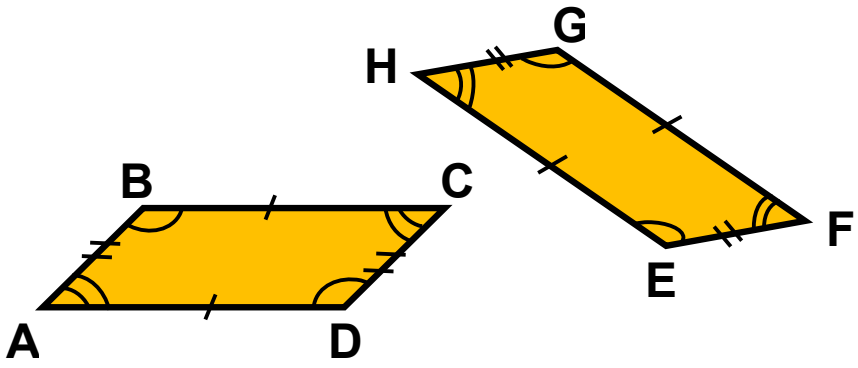
Congruent Figures

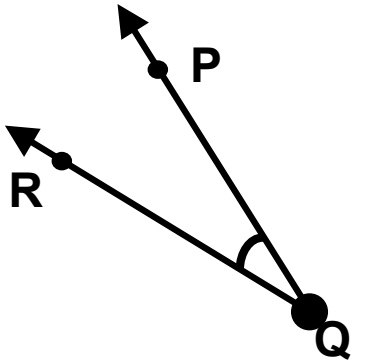
have exactly the same shape and size



≅







$\angle BAC \cong \angle PQR$

$\square ABCD \cong \square HGFE$

Complementary Angles

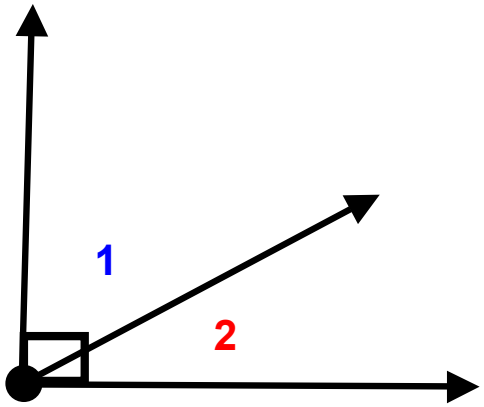


Fig 1

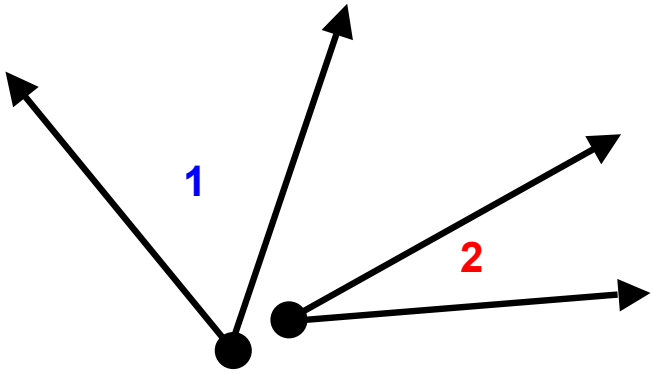


Fig 2

$m\angle 1 + m\angle 2 = 90°$

in each figure

Supplementary Angles

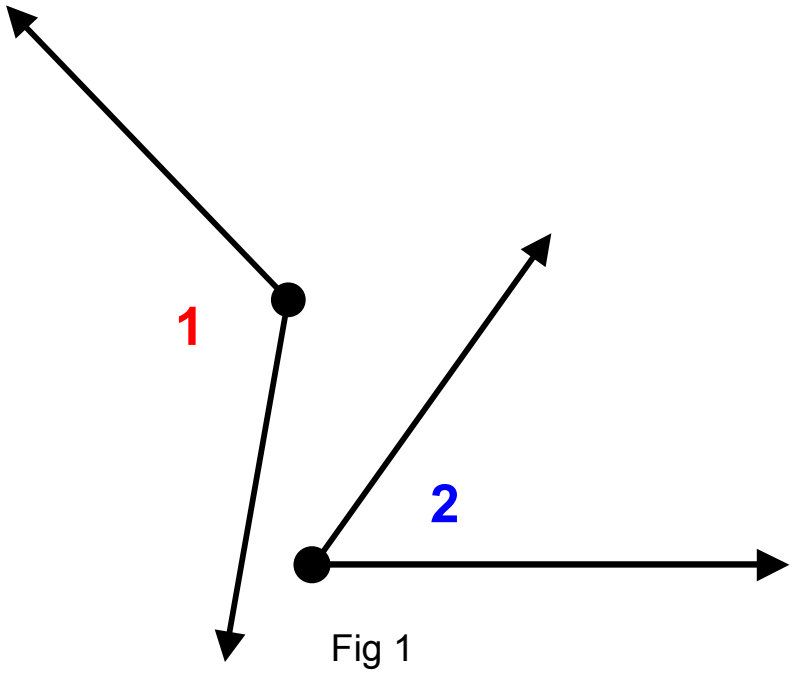


Fig 1

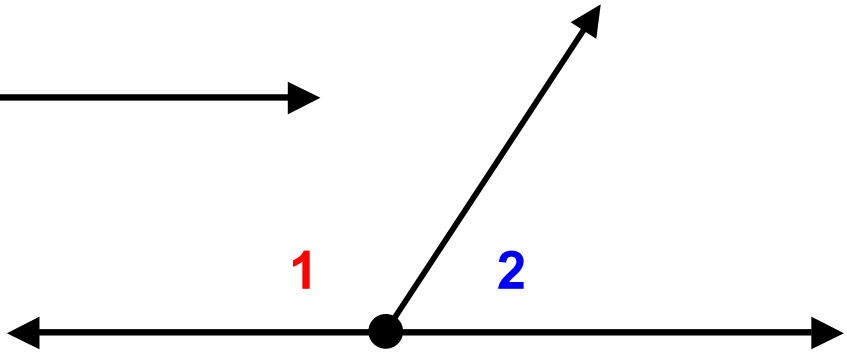
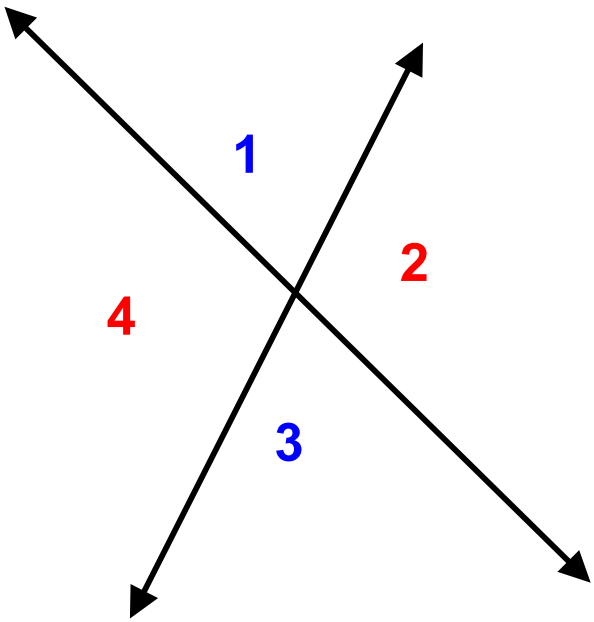


Fig 2

$m\angle 1 + m\angle 2 = 180°$

in each figure

Vertical Angles



$\angle 1$ and $\angle 3$ are vertical angles.
$\angle 2$ and $\angle 4$ are vertical angles.

$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

Adjacent Angles

∠1 is adjacent to ∠2 in each figure

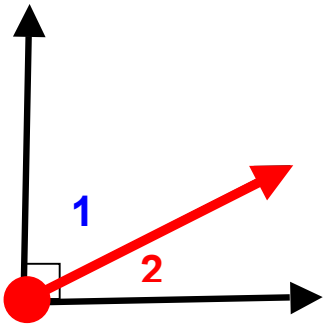


Fig 1

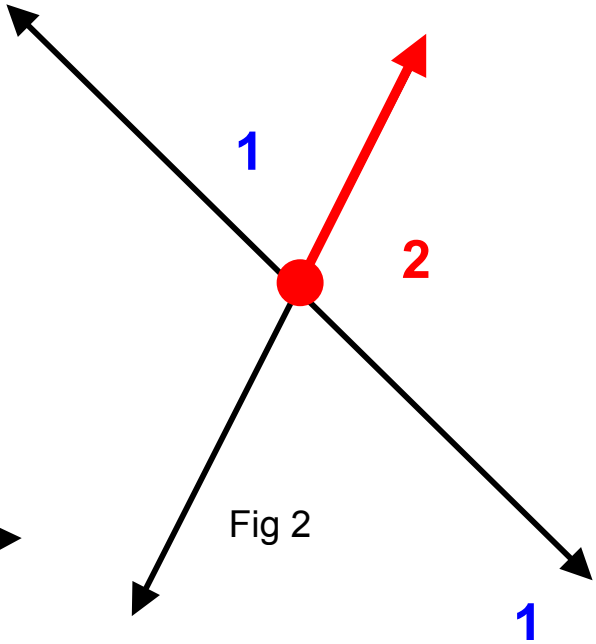


Fig 2

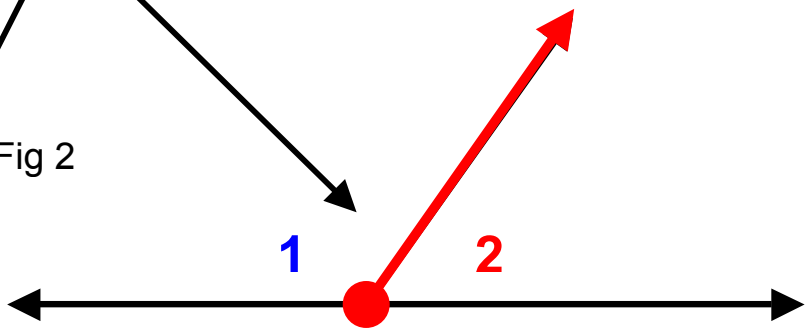
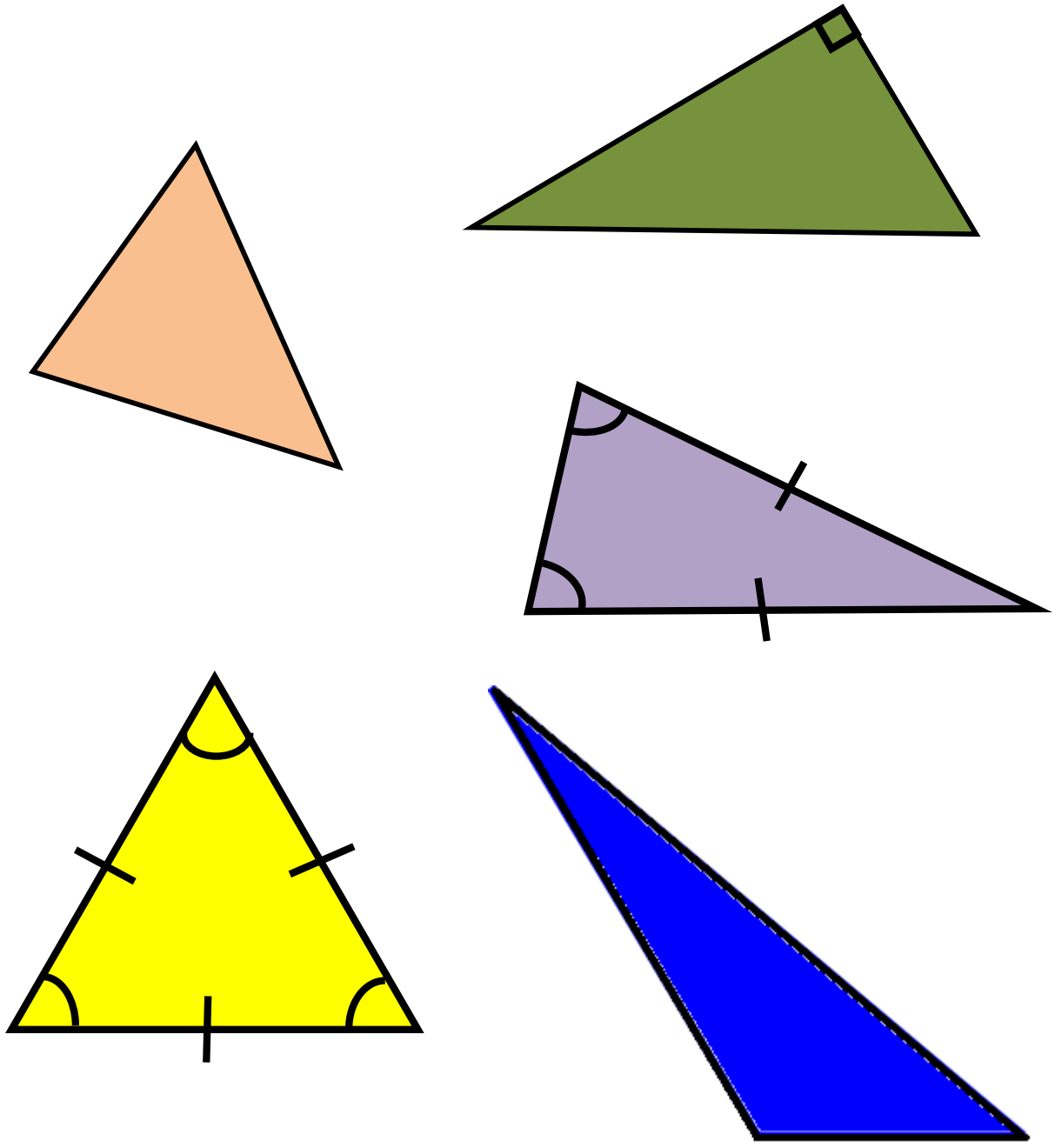


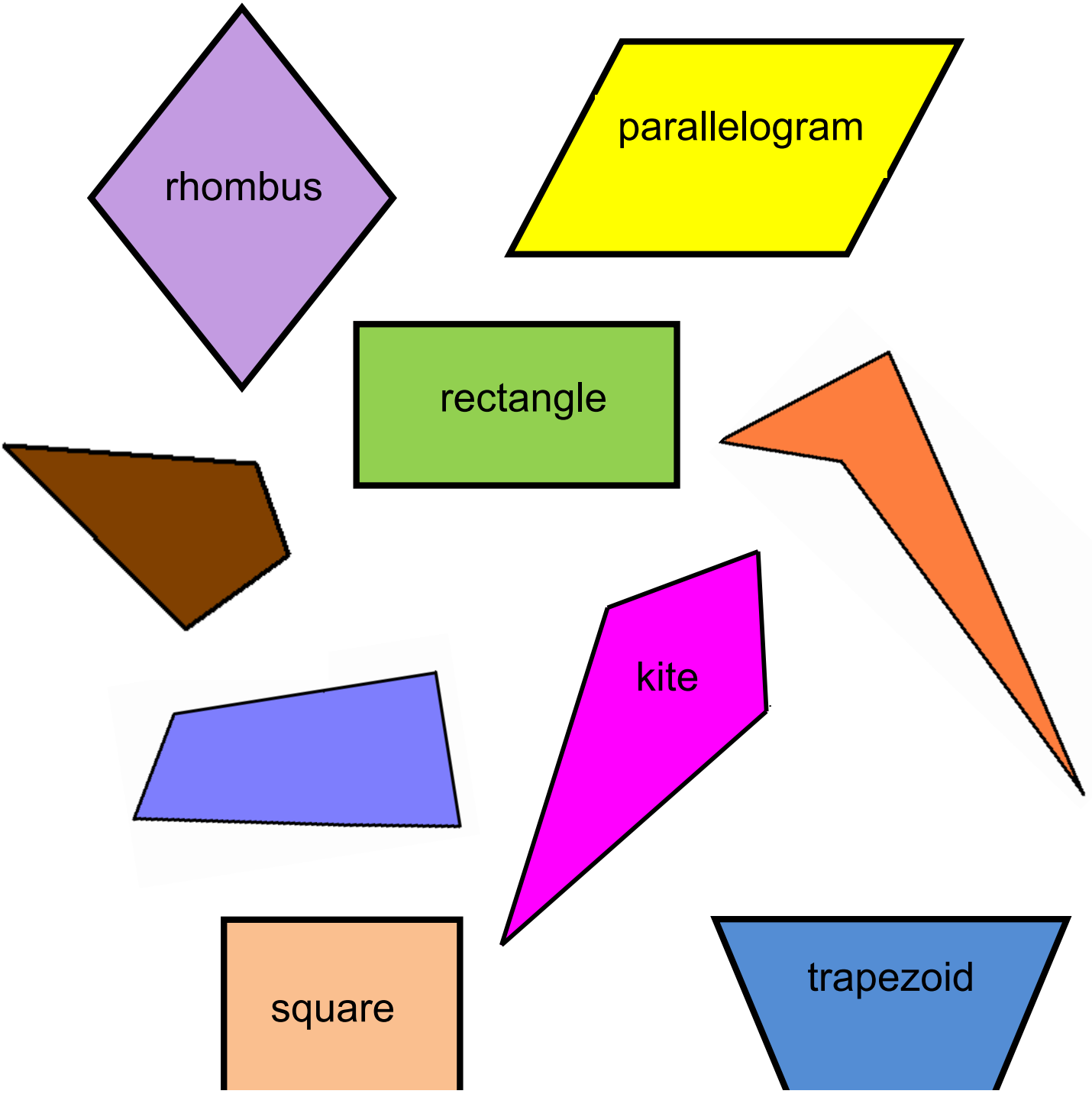
Fig 3

Share a common side and a common vertex

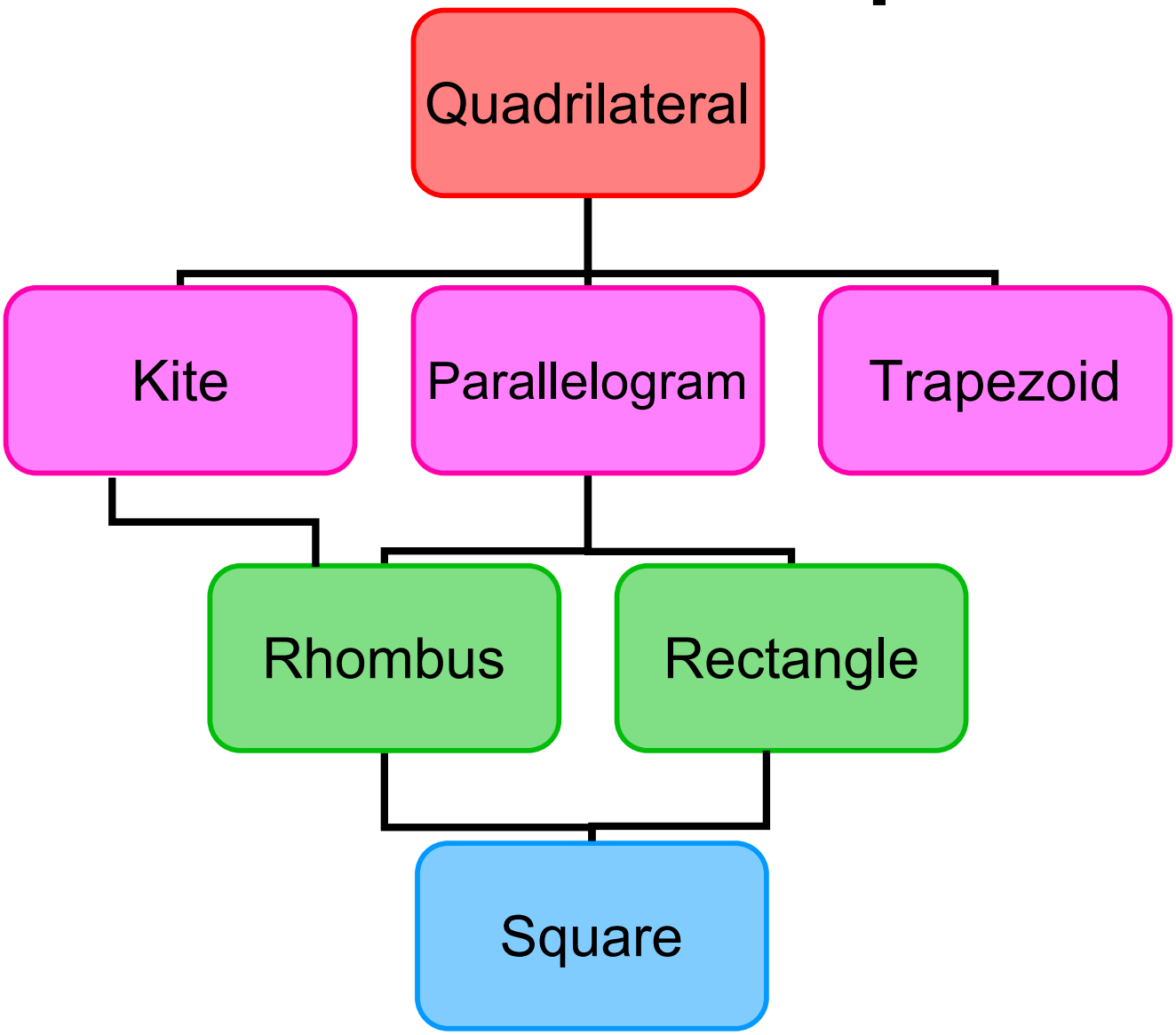
Triangles



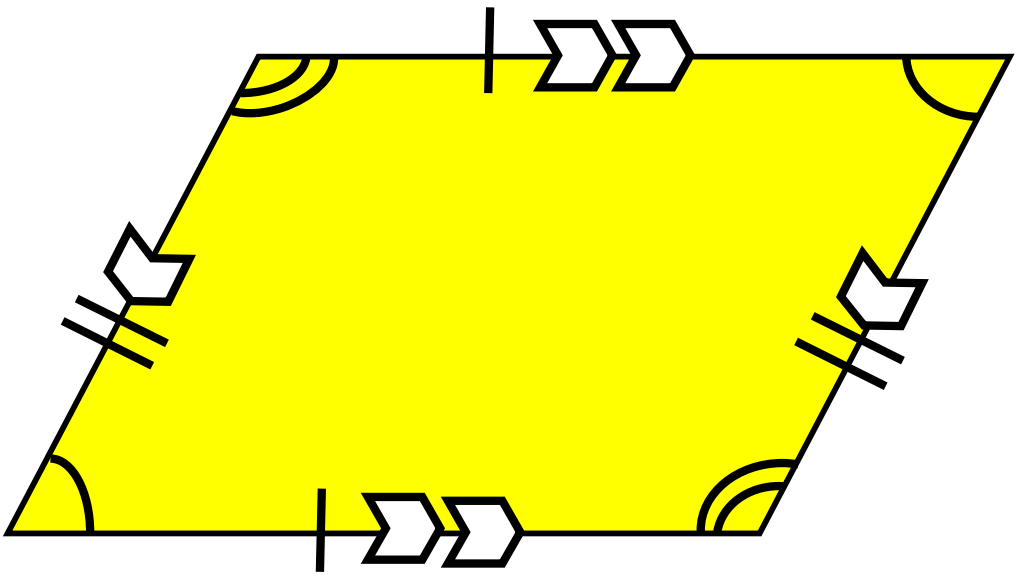
Quadrilaterals



Quadrilaterals Relationships

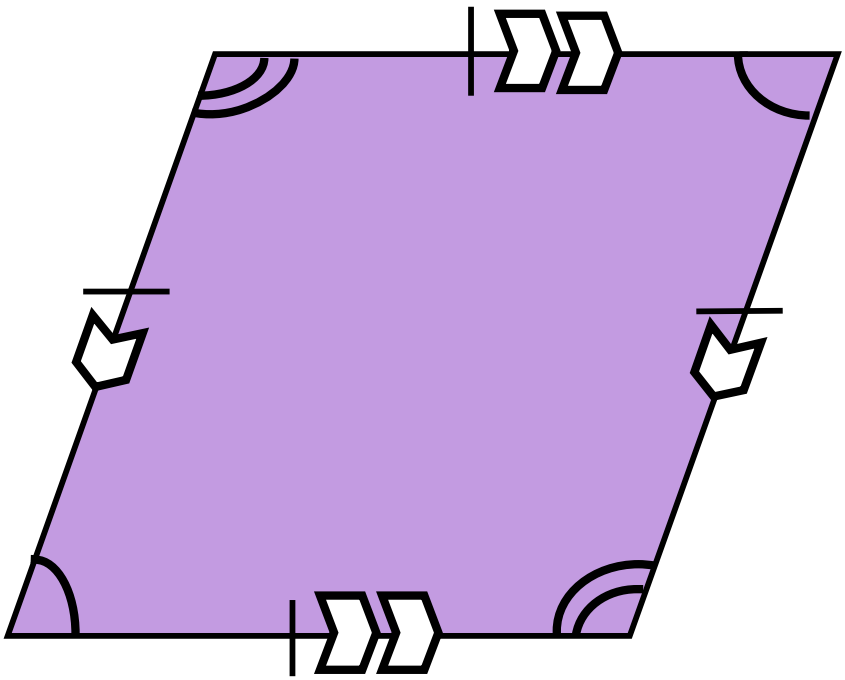


Parallelogram



- opposite angles are congruent
- 2 pairs of parallel sides
- 2 pairs of opposite sides congruent

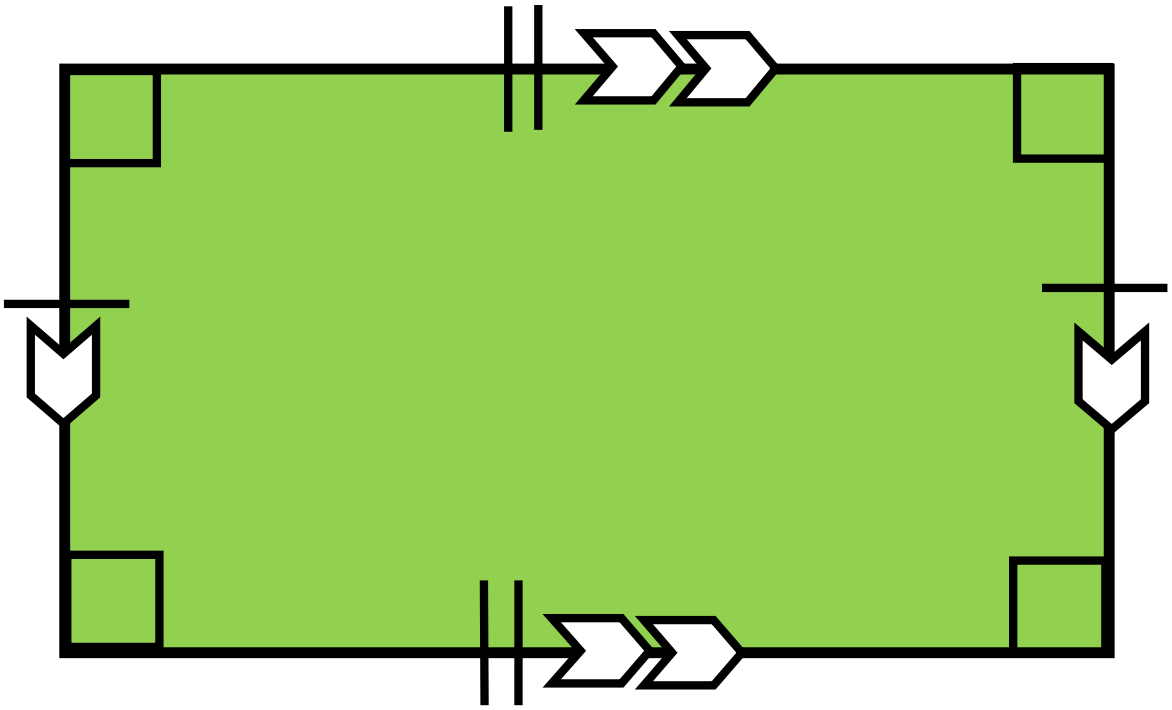
Rhombus



- opposite angles are congruent
- 2 pairs of parallel sides

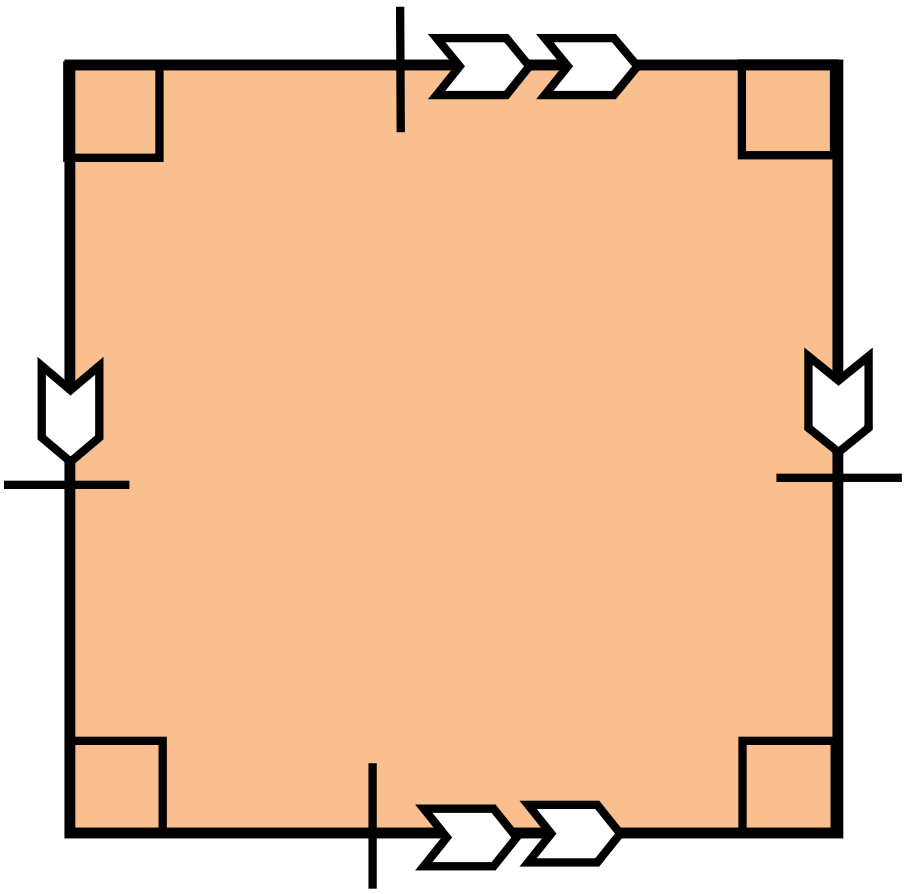
- 4 congruent sides

Rectangle



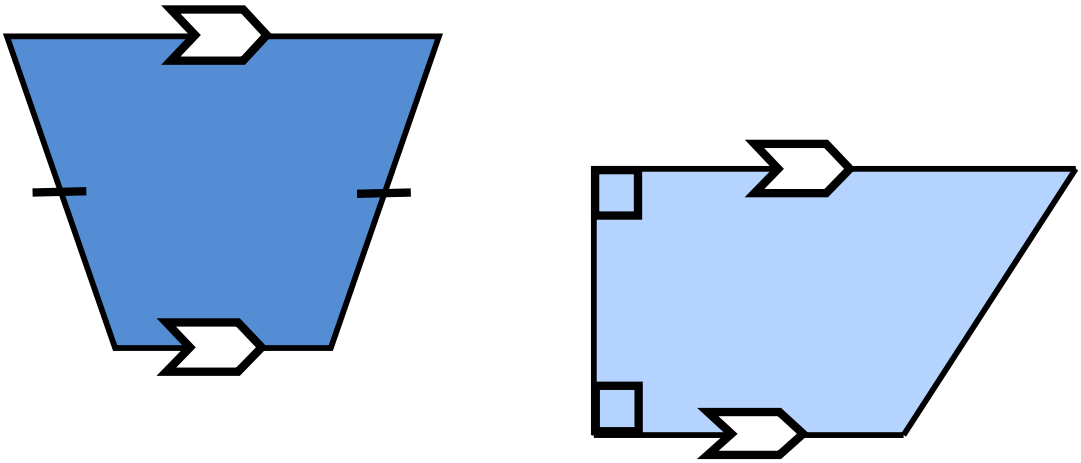
- 4 right angles
- 2 pairs of parallel sides
- 2 pairs of opposite sides congruent

Square



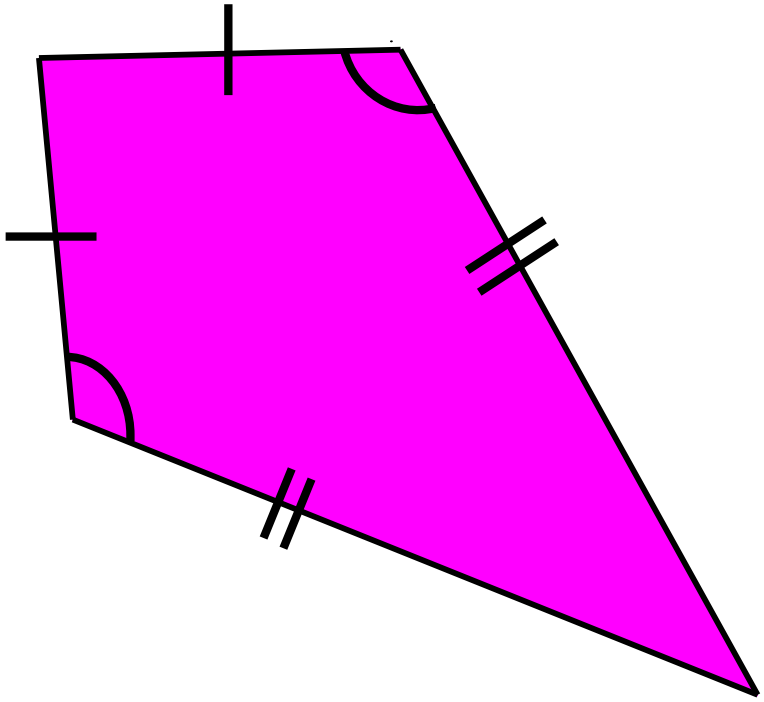
- 4 right angles
- 2 pairs of parallel sides
- 4 congruent sides

Trapezoid



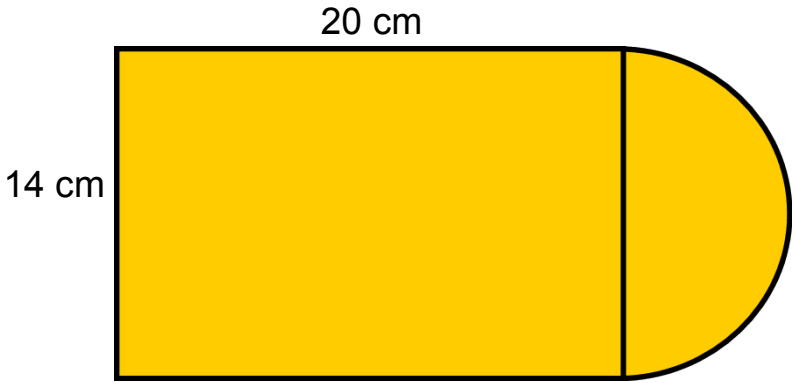
- may have zero or two right angles
- exactly one pair of parallel sides
- may have one pair of congruent sides

Kite

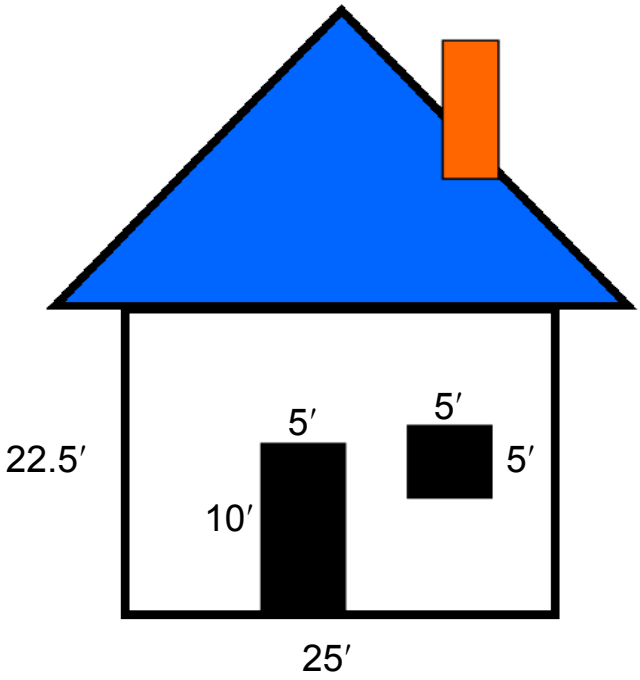


- one pair of opposite congruent angles
- 2 pairs of adjacent congruent sides

Composite Figures

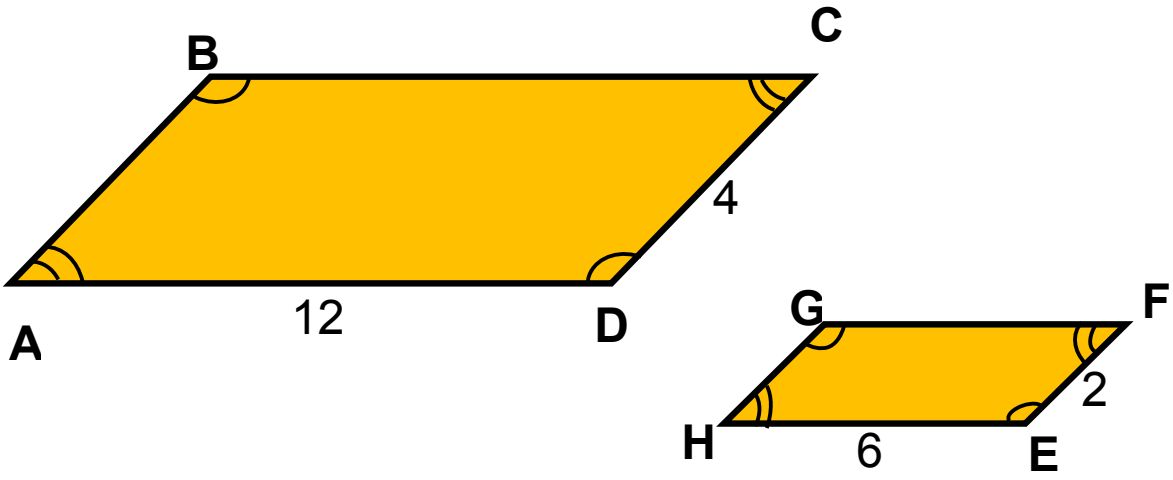


Subdivide into other figures then determine the perimeter.



Subdivide into other figures then determine the area.

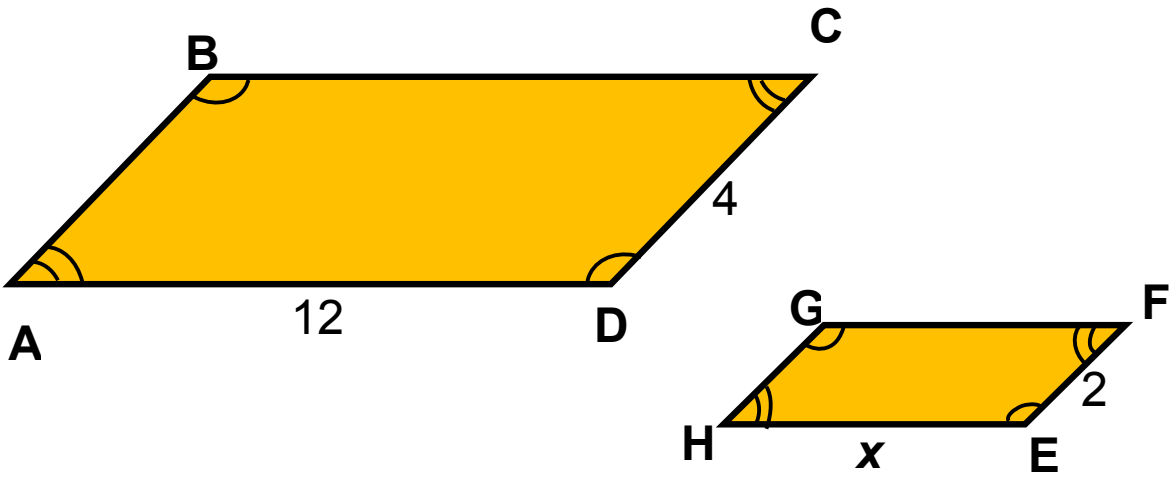
Similar Figures



ABCD ~ HGFE	
Angles	**Sides**
∠A corresponds to ∠H	AB corresponds to HG
∠B corresponds to ∠G	BC corresponds to GF
∠C corresponds to ∠F	CD corresponds to FE
∠D corresponds to ∠E	DA corresponds to EH

Corresponding angles are congruent.
Corresponding sides are proportional.

Similar Figures and Proportions

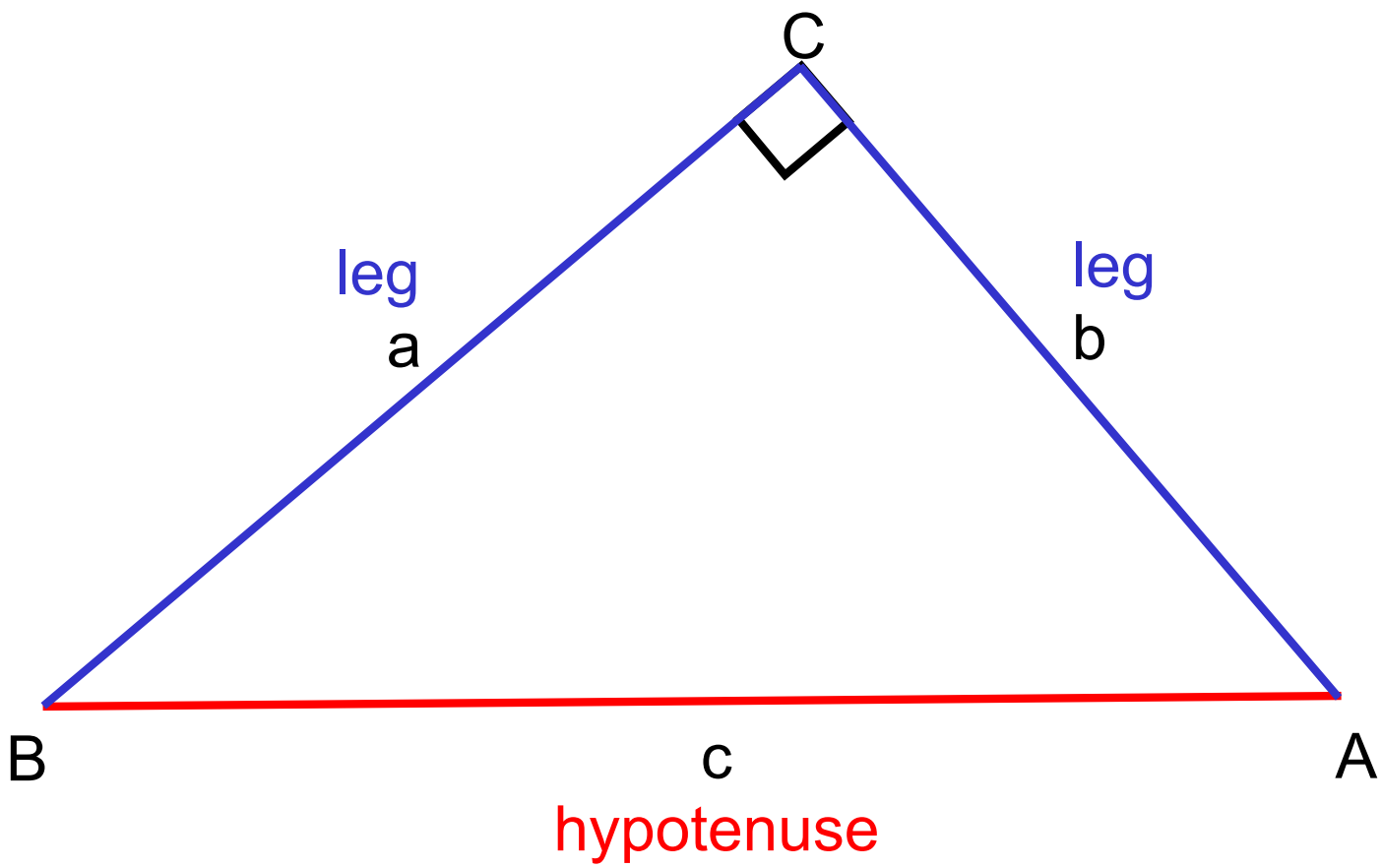


ABCD ~ HGFE

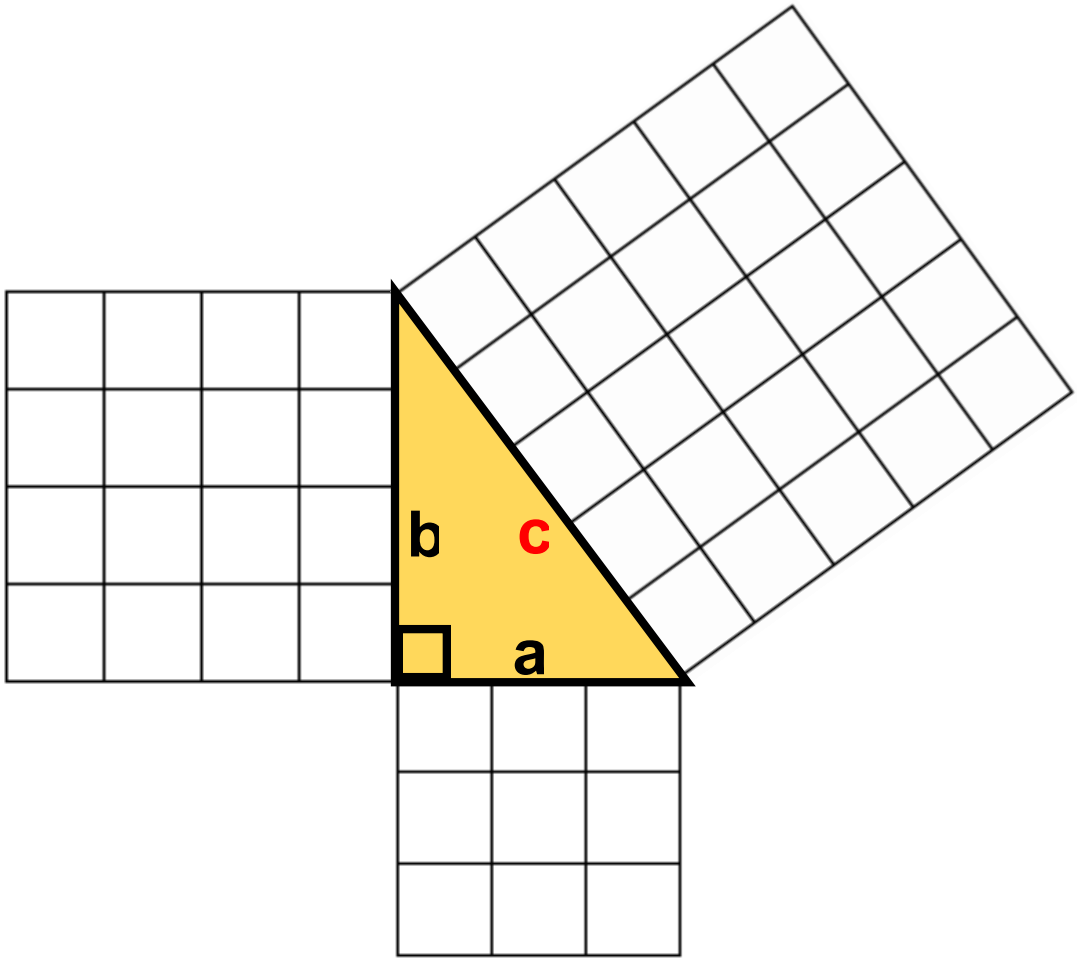
DCEF = ADHE

$42 = 12x$

Right Triangle

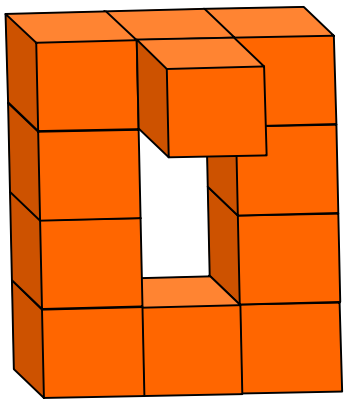


Pythagorean Theorem

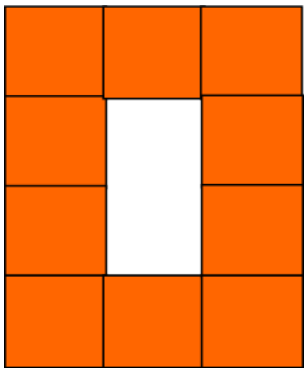


$$a^2 + b^2 = c^2$$

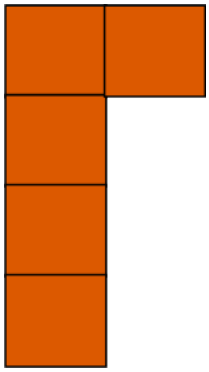
Three Dimensional Models



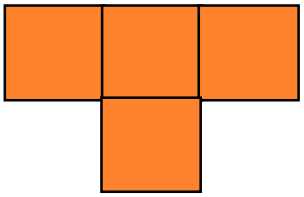
front

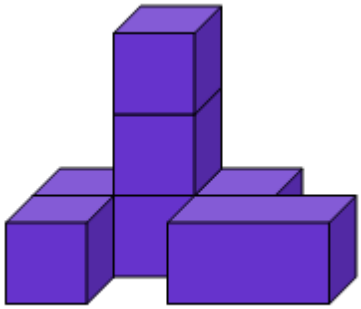


side

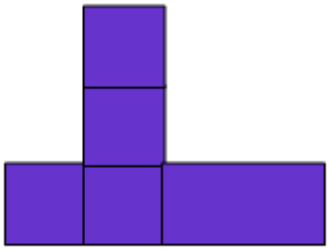


top

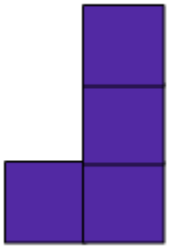




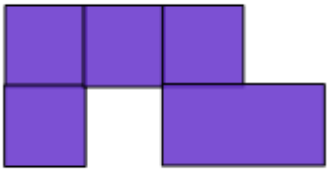
front



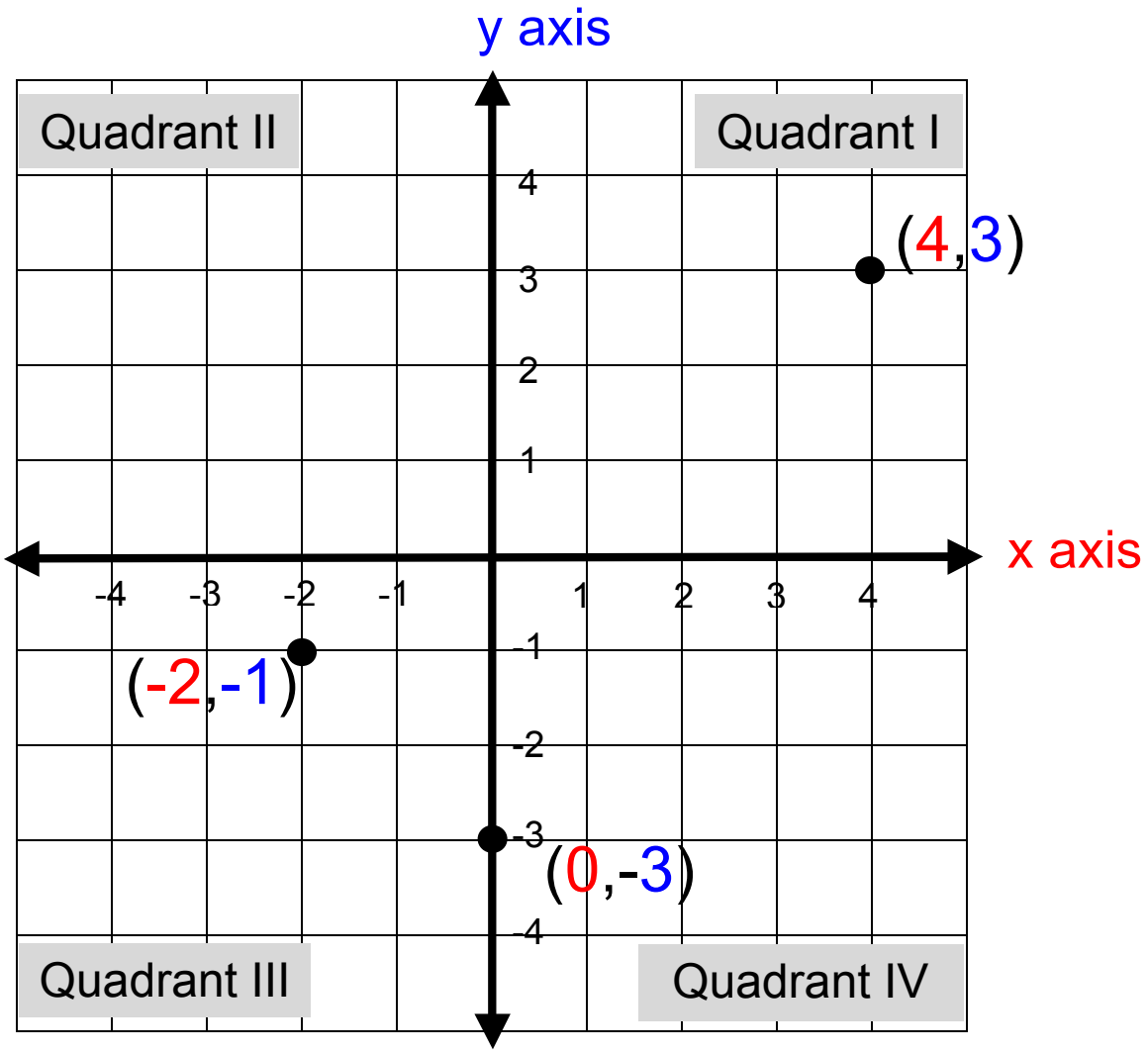
side



top

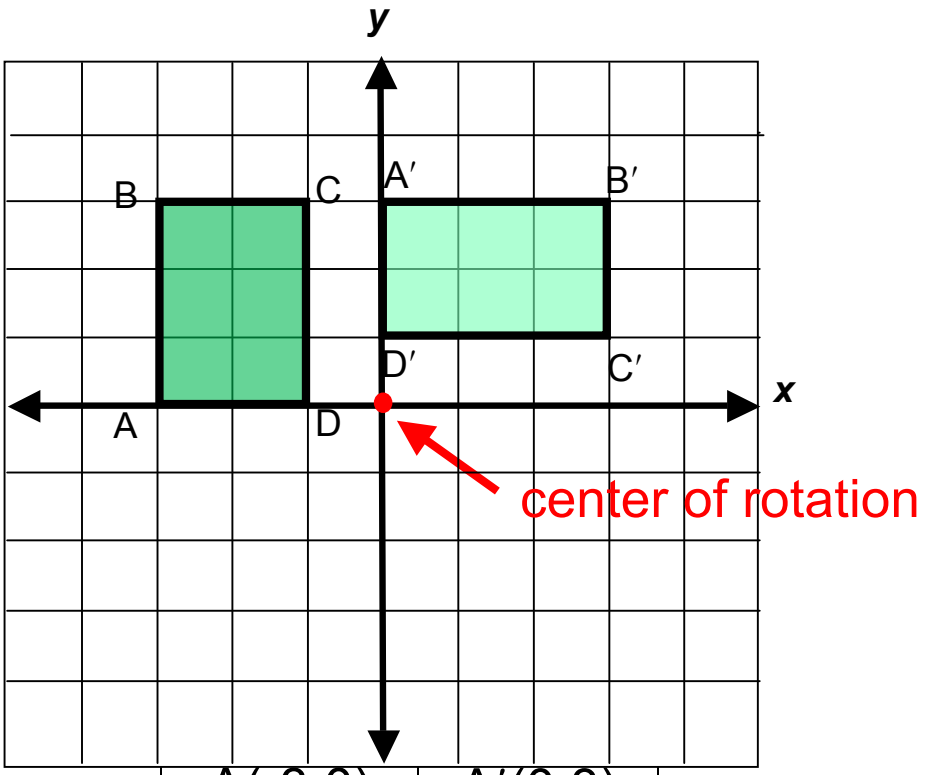


Coordinate Plane



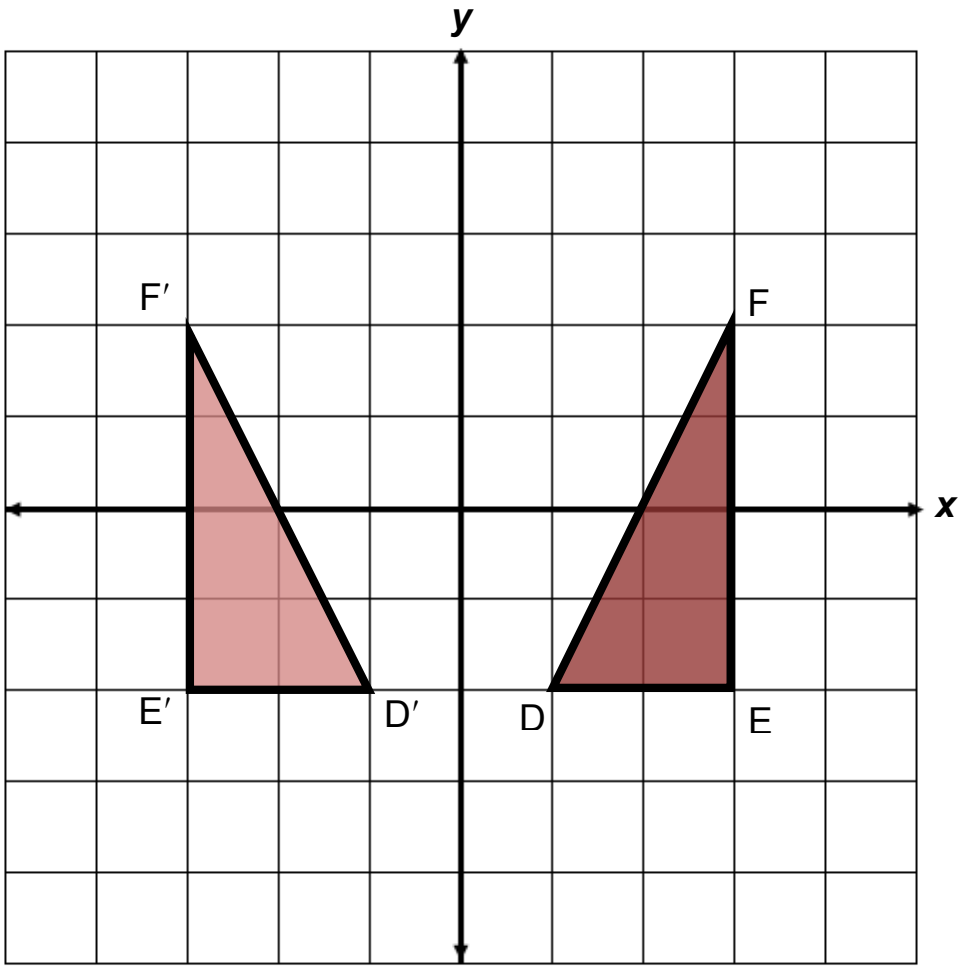
ordered pair (x,y)

Rotation



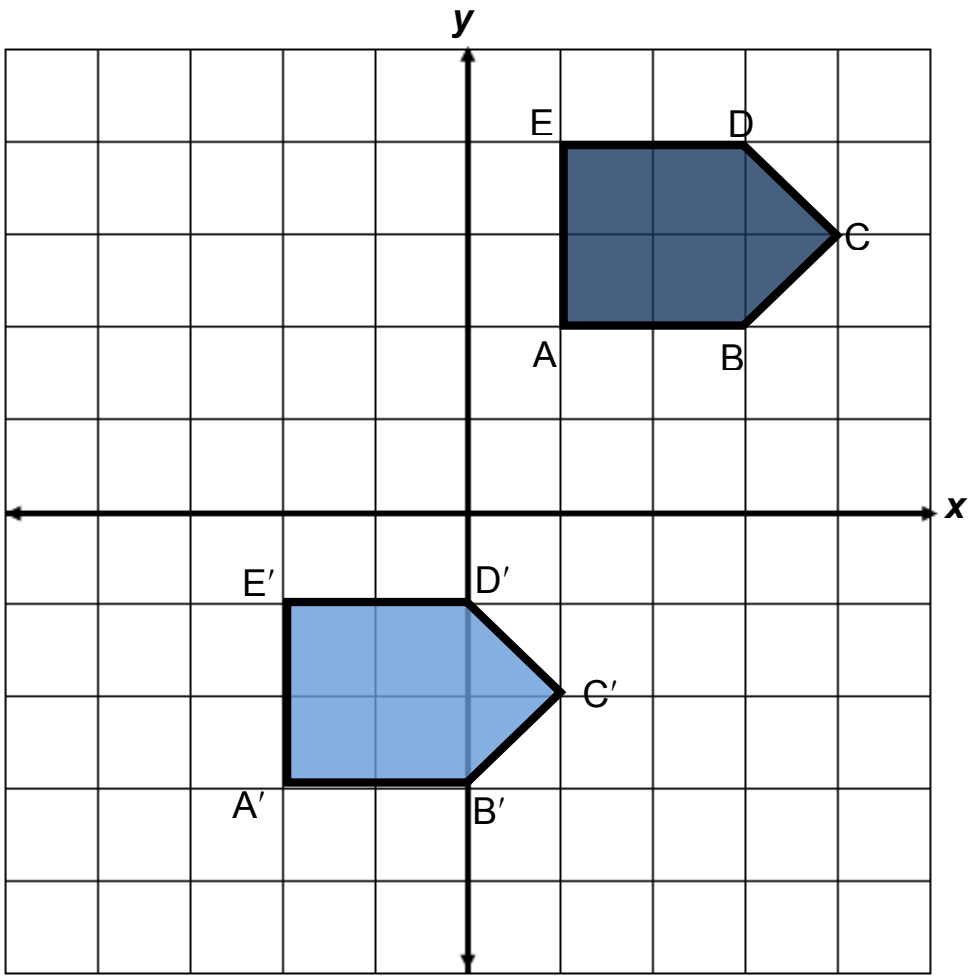
A(-3,0)	A′(0,3)
B(-3,3)	B′(3,3)
C(-1,3)	C′(3,1)
D(-1,0)	D′(0,1)

Reflection



Preimage	Image
D(1,-2)	D′(-1,-2)
E(3,-2)	E′(-3,-2)
F(3,2)	F′(-3,2)

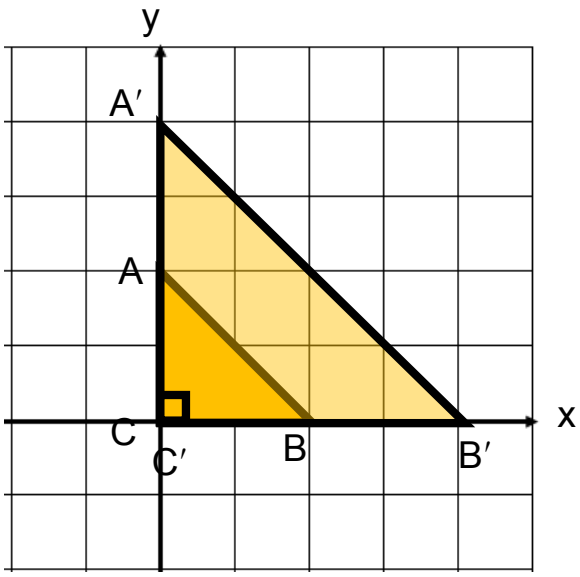
Translation

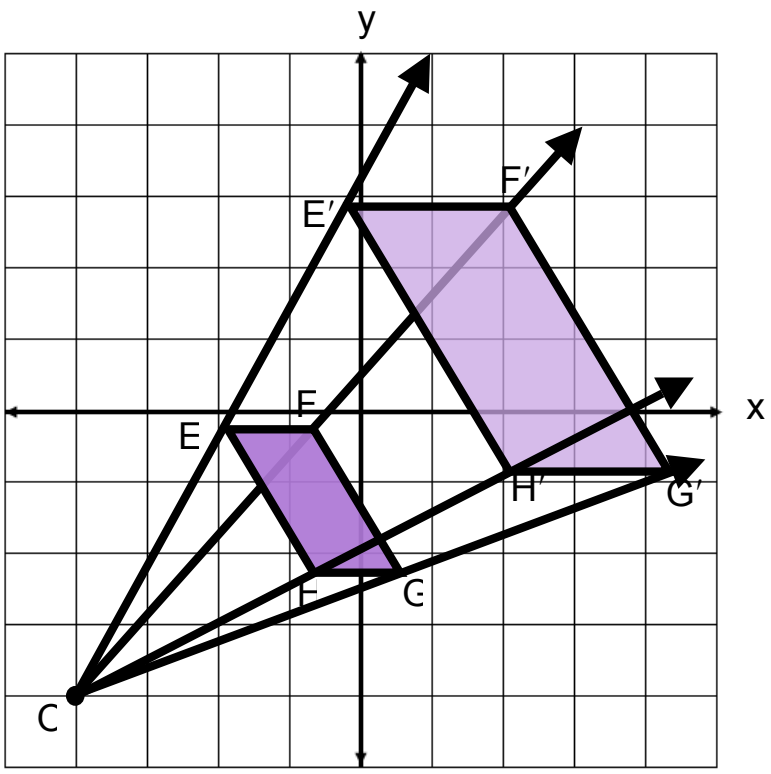


Preimage	Image
A(1,2)	A′(-2,-3)
B(3,2)	B′(0,-3)
C(4,3)	C′(1,-2)
D(3,4)	D′(0,-1)

E(1,4)	E′(-2,-1)

Dilation





Additive Identity Property

$0.3 + 0 = 0.3$

$0 + (-7) = -7$

$47 = 0 + 47$

$w + 0 = w$

Additive Inverse Property

$1.4 + (-1.4) = 0$

$(-9) + 9 = 0$

$0 = 47 + (-47)$

$x + (-x) = 0$

Associative Property

Addition:

$$(4 + 2) + 8 = 4 + (2 + 8)$$

$$x + (3x + 12) = (x + 3x) + 12$$

Multiplication:

$$(3 \cdot 1.5) \cdot 6 = 3 \cdot (1.5 \cdot 6)$$

$$2(3x) = (2 \cdot 3)x$$

Commutative Property

Addition:

$$2.76 + 3 = 3 + 2.76$$

$$(a + 5) + 7 = (5 + a) + 7$$

Multiplication:

$$-8 \cdot 23 = 23 \cdot (-8)$$

$$y \cdot 9 = 9y$$

Multiplicative Identity Property

$9 \cdot 1 = 9$

$1 \cdot (-10) = -10$

$32 = 32 \cdot 1$

Multiplicative Inverse Property

$2 \cdot \frac{1}{2} = 1$

$1 = (-19) \cdot -9$

$x \cdot \frac{1}{x} = 1 \ (x \neq 0)$

Multiplicative Property of Zero

$0 = 8 \cdot 0$

$0(-13) = 0$

$56x \cdot 0 = 0$

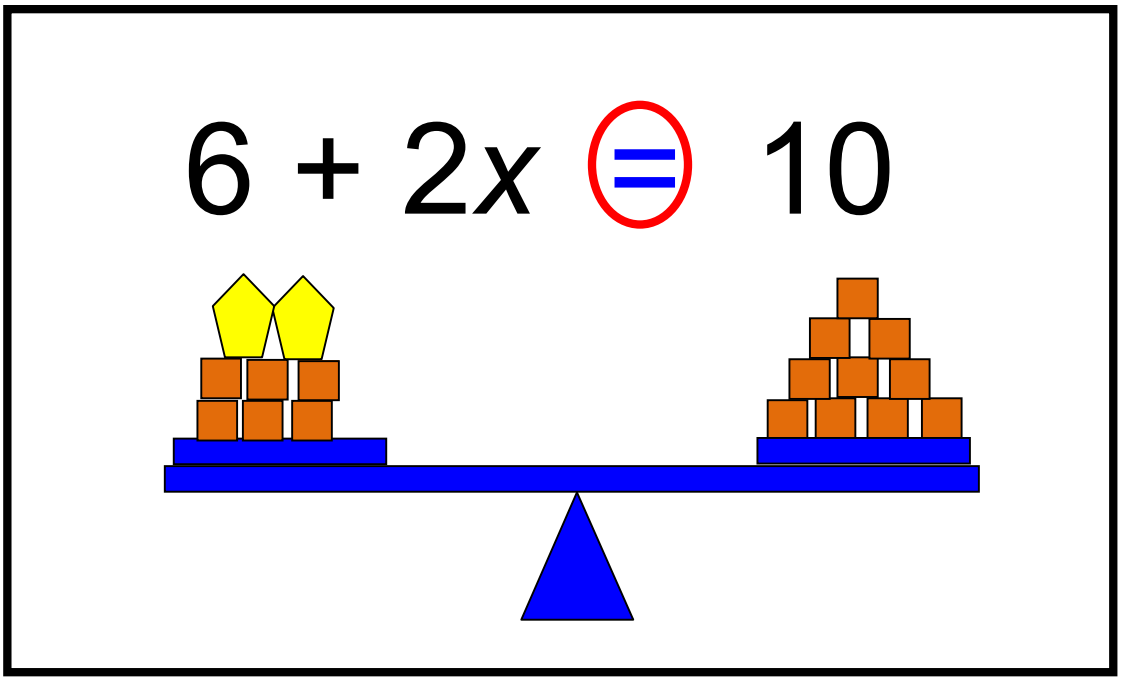
Distributive Property

$$-4(2 + 3) = -4(2) + -4(3)$$

$$5 \cdot (y - 7) = (5 \cdot y) - (5 \cdot 7)$$

$$(2 \cdot 13) + (2 \cdot 5) = 2(13 + 5)$$

Equation



A mathematical sentence stating that two expressions are equal.

$2.76 + 3 = 3 + 2.76$

$3x = 6.9$

Expression

x

-26

$2x + 3^4$

$3(y + 3.9) - 89$

Variable

$2(y + 3)$

$3 + x = 2.08$

$A = \pi r^2$

Coefficient

$(-4) + 2x$

$-7y^2$

$23ab - 12$

Term

$3x + 2y - 8$

3 terms

$-5x^2 + (-2x)$

2 terms

$23ab$

1 term

Constant

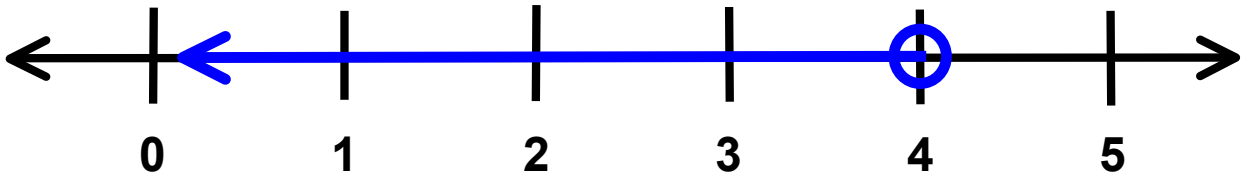
$4x - 12$

$7 - 2y + x - 6x^2$

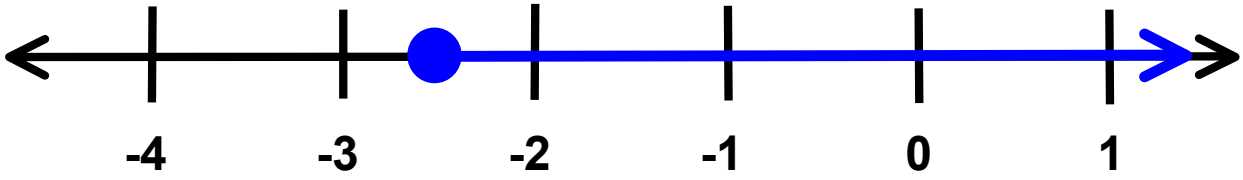
$3(x + 3.9) - 89$

Inequality

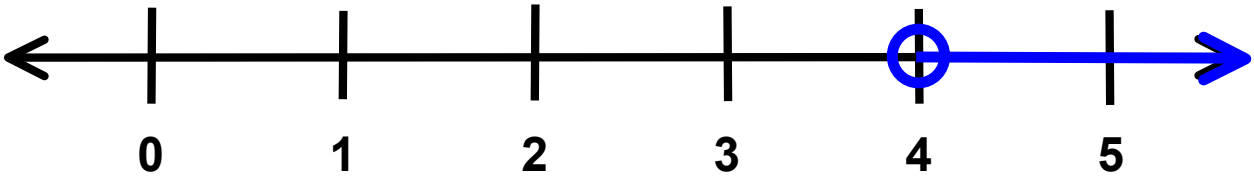
$y < 4$



$3r \geq -7.5$



$-3(n - 4) < 0$



Like Terms

$4x - 3y + 6x - 7$

$2y^2 - 3y + 7y^2$

$-5r^2 - 6 + 2r + 2$

Relations

{(2,3), (4,1), (2,5)}

x	*y*
2	2
-3	4
5	-1
0	4
1	-6

{(0,4), (0,3), (0,2), (0,1)}

Functions

{(2,4), (3,2), (0,2), (-1,2)}

x	*Y*
3	2
2	4
0	2
-1	2

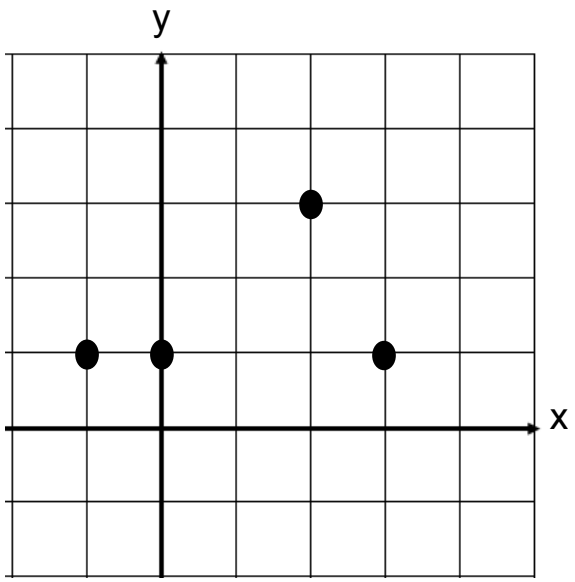


Table of Values

x	***y***
0	1
1	2
2	5
3	10
4	17

a	1	2	3	4
b	22,500	22,000	21,500	21,000

Domain

{(-2,0), (-1,1), (0,2), (1,3)}

x	*Y*
-2	0
-1	1
0	2
1	3

{-2,-1, 0, 1}

Range

{(-2,0), (-1,1), (0,2), (1,3)}

x	*y*
-2	0
-1	1
0	2
1	3

{0, 1, 2, 3}

Dependent/ Independent Variable

Determine the distance a car will travel going 55 mph.

$d = 55h$

independent

h	d
0	0
1	55
2	110
3	165

dependent

Independent Variable

$y = 2x + 7$

x represents the independent variable (input values or domain)

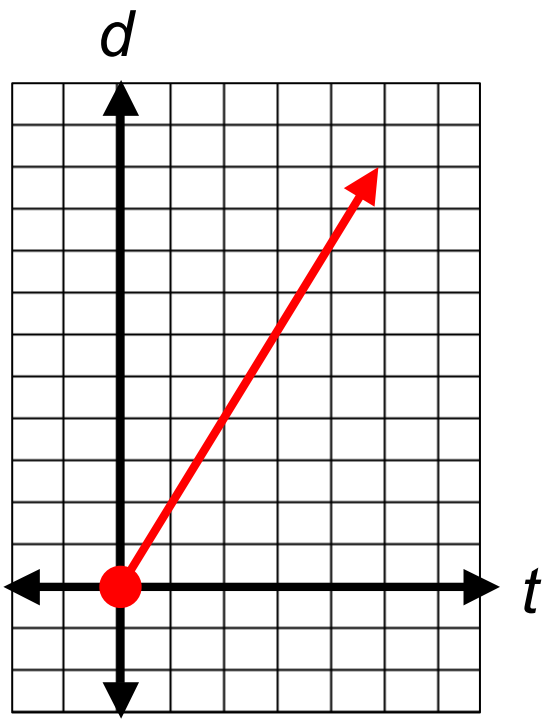
Dependent Variable

$y = 2x + 7$

y represents the dependent variable (output values or range)

Connecting Representations

The total distance Sam walks depends on how long he walks. If he walks at 2.1 mph, show multiple representations of the relationship.



t	d
0	0
1	2.1
2	4.2
4	8.4

$$d = 2.1t$$

Multistep Equations

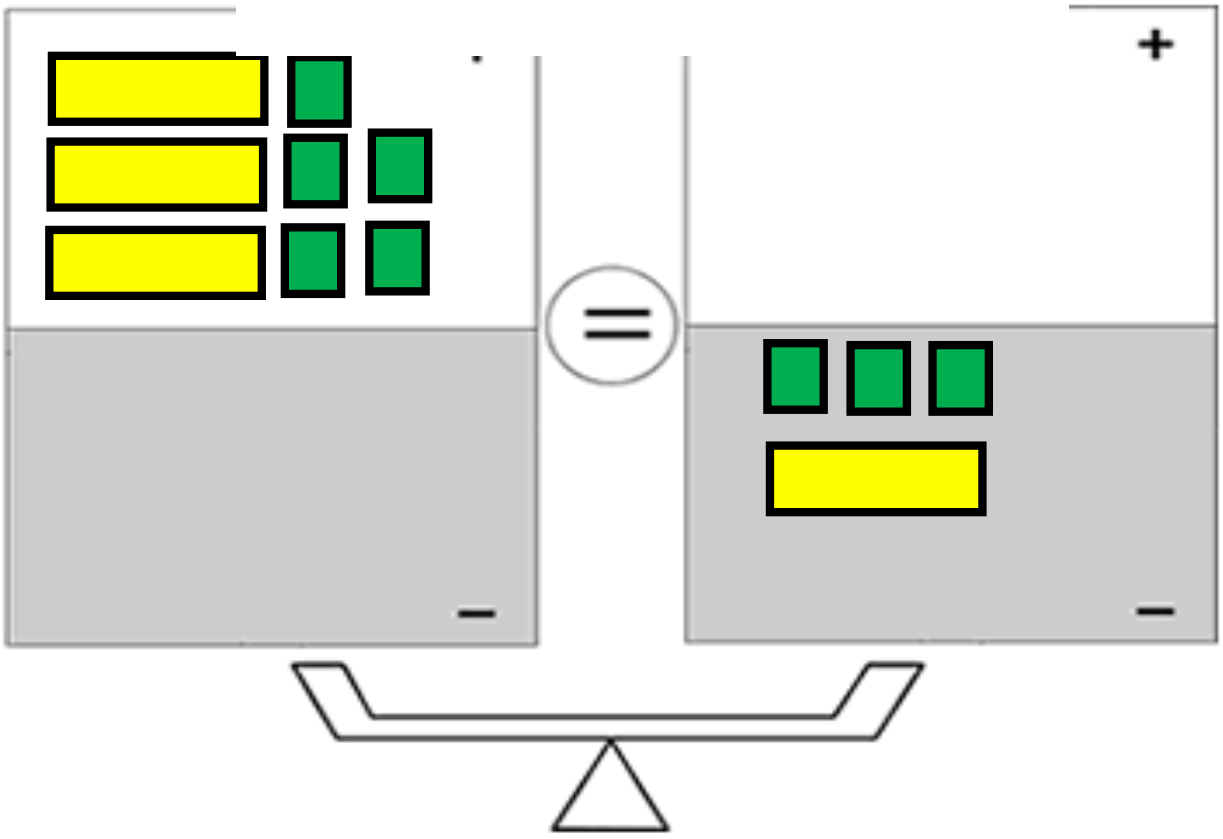
$2x - 5.7 = -3.4x + 11.04$

$23\ (n + 9) = -\ 56n$

$25 = 6p\ -\ 5\text{-}4$

Multistep Equation

$$3x + 5 = -3 - x$$



Unit Rate as Slope

A student walks 2 miles per hour

$$\frac{\text{2 miles}}{\text{1 hour}}$$